Discussion of the application of rate dependent compaction models to the Groningen gas field

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General Introduction

This document contains studies exploring the rate-dependent compaction models for reservoir rock (Ref. 1 to 2). This model has most recently been documented in a report by TNO (Ref. 3 and 4), and in the article by Pruiksma et al., (2015, Ref. 5).

This document contains discussions of this model by:

- Florian Lehner,
- Teng Fong Wong,
- Anthony Mossop,
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- Chris Spiers,
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The rate-dependent compaction model has proven to be of value in forecasting of subsidence and is used by NAM for forecasting of subsidence above the gas fields in the north Netherlands (Ref. 6). These studies primarily address the derivation of this model from primary principles and its applicability for forecasting surface subsidence and seismicity above gas-fields.

Rate-dependent compaction models have recently become an area of interest for the forecasting of the decline in seismicity in the Groningen gas field, during the pressure equilibration phase and after gas production from the field was ceased (Ref. 7). The studies discussing and reviewing the RTCM-model contained in this document take different perspectives at evaluating whether the model has robust theoretical and/or experimental foundations, and how it can be used for field predictions.

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Title	Review of rate dependent compaction models for reservoir rock	**Date**	1 Augustus 2024
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Place in the Study and Data Acquisition Plan	<u>Study Theme</u>: Geomechanics <u>Comment</u>: This document contains studies exploring the rate-dependent compaction models for reservoir rock. This model has most recently been documented in a report by TNO, and in the article by Pruiksma et al. (2015). This document contains discussions of this model by: • Florian Lehner, • Teng Fong Wong, • Anthony Mossop, • Peter Schutjes, • Chris Spiers, • Mateo Acosta and Jean-Philippe Avouac The rate-dependent compaction model has proven to be of value in forecasting of subsidence and is used by NAM for forecasting of subsidence above the gas fields in the north Netherlands. These studies primarily address the derivation of this model from primary principles and its applicability for forecasting surface subsidence and seismicity above gas-fields. Rate-dependent compaction models have recently become an area of interest for the forecasting of the decline in seismicity in the Groningen gas field, during the pressure equilibration phase and after gas production from the field was ceased. The studies discussing and reviewing the RTCM-model contained in this document take different perspectives at evaluating whether the model has robust theoretical and/or experimental foundations, and how it can be used for field predictions.		
Directly Linked research	(1) Forecasting of Subsidence. (2) Induced seismicity.		
Used Data	Several documents discussion the RTCM model.		
Assurance			

An Evaluation of Report TNO 2013 R11405

prepared for

Fircroft Engineering Services BV, Rijswijk

on behalf of

Shell/NAM Development Groningen

by

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Summary

The present report was prepared at the request of Shell/NAM Development Groningen. It contains detailed technical discussions of the reservoir compaction models studied in report TNO 2013 R11405 by Pruiksma *et al.* [1], with a view on their potential use for predicting the compaction behaviour of a sandstone of the type encountered in Groningen reservoir. Each model is evaluated against a background of known or expected rock-mechanical behaviour and the theoretical needs to be met by any model that aims at the prediction of depletion-induced surface subsidence. In two cases, concerning the SLS model and the RTCM, a comparison could be made with experimental observations on an Oldorp 1-1 core sample, from which additional insight into the potential and limitations of these model was obtained.

The principal conclusions of this evaluation are the following:

- The Soft soil models discussed in [1] fail to describe the compaction behaviour of consolidated sandstones.

- If replaced by a more general 3D poroviscoelastic model, the SLS model would be an obvious first step beyond poroelasticity in allowing for rate-dependent behaviour. Appropriate poroviscoelastic solutions of the compaction/subsidence problem in 3D exist already in the literature, or could be obtained by a relatively small effort, and should be compared with field data.

- The lack of experimental support and of a theoretical base for the extrapolation from laboratory to field conditions suggests that the RTCM should be restricted in its use to the correlation/extrapolation of field data among reservoirs with a substantial depletion history.

- 3D FE formulations for complex elasto-plastic, rate-dependent models exist an should be used to explore several open questions, among which the question of a subsidence delay effect that may originate in the reservoir surroundings, in particular in evaporites, and possibly even in the uppermost soft sediments.

- There exist several recent experimental studies of the creep response of reservoir rocks that highlight the importance of temperature and 3D loading path effects, in particular for creep rates. Contact should be made with these studies, as these are more in line with the intended applications in Groningen and other fields than applications to shallow foundations on soft soils that form the background of Report TNO 2013 R11405.

Contents

Introduction

This report contains an evaluation of report TNO 2013 R11405 by Pruiksma *et al.* [1], of its scientific content and technical significance as a 'tool box' for the prediction of surface subsidence displacements that are caused by reservoir compaction. To be able to fulfill this task, it was soon found necessary to carry out a fairly detailed technical analysis of the contents of the report, essentially a technical discussion of four different theoretical candidate models for the predicting the uniaxial compaction of a Groningen reservoir rock in response to changes in an effective axial stress. Four models that share two characteristics, i.e., that the deformation remains uniaxial and governed by a single component of stress, and secondly, that a this uniaxial deformation exhibits a rate-dependence of one or the other form. Rate-dependent behaviour remains in fact the subject of principal interest throughout the report, motivated by a need to explain and hopefully model the delayed surface subsidence (delayed w.r. to the drop in mean reservoir pressure) observed at Groningen and in other gas fields of the Northern Netherlands.

In the following we will discuss these models one by one, point out its principal features and limitations, and also consider the question of how to interpret experimental data in terms of one or the other theoretical model. The last step is of course an essential one, but also one that is barely dealt with by Pruiksma *et al.*. Here we were able to make use of a data set provided by D. Doornhof of NAM. One of the shortcomings that Pruiksma *et al.* had apparently to deal with is in fact the scarcity of experimental data and the unfortunate tendency of the original experimentalist to vary conditions during an experiment that should have been kept constant. As a result there are certain critical theoretical features which–as we shall see–have never been confirmed directly by the necessary experiments. This concerns in particular the exact shape of the so-called isotachs, i.e., plots of strain vs. stress at constant strain- or stress rates, since the preference of the authors for one or the other model turns out to be determined mostly by these shapes, that is by shapes that apparently have never been determined directly.

The isotach models described by Pruiksma *et al.* originate in soil mechanics. This may explain the exclusive concern with uniaxial, oedometer-type deformation and the fitting of such data by theoretical predictions. The influence of soil behaviour is most apparent with the first two isotach models that are discussed in [1], viz. the Soft Soil Isotach Model and the Stress-linearized Isotach model. It turns out that these models predict a type of “stress-free creep” behaviour that is unlikely ever to be observed in consolidated reservoir rocks. Models of this kind clearly will not meet the minimal requirements of a reservoir compaction model.

The third of the four models discussed by the authors is that of a Standard Linear Solid (SLS). This represents the simplest rheological model of a linear viscoelastic solid. Models of this type have long been in use in many engineering disciplines and in soil mechanics in particular, although often in the form of more complex arrangements of springs and dash-pot in parallel and in series. Strength elements are occasionally added to these models to endow them with an inelastic yield behaviour. These models are conceptually clear and are easily presented in a form suitable for 3D applications. Pointing in the direction of future theory developments, with one or two examples, we note that these rheological models also lend themselves to interpretations in terms of more sophisticated internal variable theories and it is from such models that one may expect progress with the development of constitutive theories constructed for a specific micro-mechanism of (rate-dependent) deformation. In this context, we therefore suggest to focus on recent developments and research sponsorship in the direction of experimentally and micromechanically informed research on the materials encountered in the field. In a first attempt to match theory and experiment, we shall consider a fit of the simple SLS model (which features only a single characteristic time) to a data set obtained in a compaction experiment on an Oldorp 1-1 core sample. The outcome of this comparison will show how rate-effects that are seen at extremely high loading rates in the laboratory may disappear under field conditions, while effects that appear in the field may never be observed in an experiment.

The Rate Type Compaction Model (RTCM) is dealt with in considerable detail. A characteristic scaling relation of this model links the rate-dependent compressibilities (or compliances) for different isotachs over an unlimited range of loading rates. Following a discussion of the basic structure of the model, we will point out a way to apply the RTCM without any appeal to a fictitious “geological loading rate”, a meaningless concept which used in different ways in [1]. A comparison with experimental results for the Oldorp 1-1 core will be shown to yield good agreement between observations and model predictions of creep strains as a function of time. Creep strains are determined from a closed-form solution of the relevant differential equation of the RTCM isotach formulation that vindicates an earlier result of de Waal [2]. But the lack of a convincing theoretical base suggest the correlation/extrapolation of field data as the most likely area of application of the RTCM.

1. Discussion of Soft Soil Isotach Model (Section 3)

The model is essentially an extension of a classical stress-strain relation of soil mechanics by a term that depends on the strain rate. The total strain $\varepsilon = \varepsilon_d + \varepsilon_s$ is composed in this model of a 'direct' (elastic) strain ε_d and a rate dependent 'secular' strain ε_s. What matters to us is the latter, i.e., in dealing exclusively with ε_s we shall be able to discuss all pertinent issues with this model. The constitutive relation for the secular strain proposed in [1] is of the following form:

$$\varepsilon_s = b \ln \frac{\sigma'}{\sigma'_{ref}} - c \ln \frac{\dot{\varepsilon}_s}{\dot{\varepsilon}_{s,ref}} \tag{1}$$

This may be rewritten as equation (8) of [1], which can obviously be brought into the form $\mathrm{d}y = f(t)\,\mathrm{d}t$ in terms of the variable $y = e^{\varepsilon_s/c}$ and a given function $f(t) = (\sigma'(t)/\sigma'_{ref})^{b/c}\dot{\varepsilon}_{s,ref}/c$ of time, whereupon it may be integrated to give[1]

$$\varepsilon_s = c \ln \left\{ 1 + \int_{t_0}^{t} f(t')\,\mathrm{d}t' \right\} \tag{2}$$

for $\varepsilon_{s,0} = \varepsilon_s(t_0) = 0$. In the special case of creep at $\sigma' = \sigma'_0$ one obtains

$$\varepsilon_s = c \ln \left[1 + \dot{\varepsilon}_{s,ref}/c \left(\frac{\sigma'_0}{\sigma'_{ref}} \right)^{b/c} (t - t_0) \right] . \tag{3}$$

while for the case of a constant loading rate, when $\sigma'(t) = \dot{\sigma}'(t - t_0)$, the strain is given by

$$\varepsilon_s = c \ln \left\{ 1 + \frac{\dot{\varepsilon}_{s,ref}}{c/\beta} \frac{\sigma'_{ref}}{\dot{\sigma}'} \left[\left(\frac{\sigma'}{\sigma'_{ref}} \right)^{\frac{1}{\beta}} - 1 \right] \right\} , \quad \text{with} \quad \beta = \frac{1}{b/c + 1} . \tag{4}$$

The question now arises as to which value to assign to the parameter $\dot{\varepsilon}_{s,ref}$. Pruiksma *et al.* classify $\dot{\varepsilon}_{s,ref}$ as a "free parameter", but leave the reader of their report in the dark about the exact meaning of this term. It is therefore necessary to consider the steps taken (or not taken) in their Section 3.3 in some detail.

We begin by observing that, with increasing stresses, the solution (4) approaches the straight-line asymptote

$$\varepsilon_s = c \left\{ \ln \left(\frac{\sigma'}{\sigma'_{ref}} \right)^{\frac{1}{\beta}} - \ln \left[\frac{c/\beta}{\dot{\varepsilon}_{s,ref}} \frac{\dot{\sigma}'}{\sigma'_{ref}} \right] \right\} . \tag{5}$$

[1] Note that this allows one to avoid the numerical integration suggested by Pruiksma *et al.* for arbitrary loading histories.

The loading rate $\dot{\sigma}'$ enters here as a curve parameter for the bundle of solutions of (4), each of which approaches its straight-line asymptote (5), as is shown in Fig. 1 of [1].

Assuming now that we are given experimental data for some constant loading rate $\dot{\sigma}'$, we can plot the strain against $\ln \sigma'$ and, supposing we find the data to exhibit a linear trend, identify this with the asymptote (5) and determine its intercept $\sigma'(0, \dot{\sigma}')$ at $\varepsilon_s = 0$. According to (5) this stress is then to be interpreted as

$$\sigma'(0, \dot{\sigma}') = \sigma'_{ref} \left(\frac{c/\beta}{\dot{\varepsilon}_{s,ref}} \frac{\dot{\sigma}'}{\sigma'_{ref}} \right)^{\beta} . \tag{6}$$

The parameter $\dot{\varepsilon}_{s,ref}$ appears here as a *fixed* rather than "free" parameter, determined by the pair of interrelated empirical quantities $\dot{\sigma}'$ and $\sigma'(0, \dot{\sigma}')$. It therefore seems reasonable to assume - as Pruiksma *et al.* do - that $\dot{\varepsilon}_{s,ref}$ remains indeed constant over a relevant range of loading rates. This will however delimit the range of validity of the constitutive model (1). Given $\dot{\sigma}'$ and $\sigma'(0, \dot{\sigma}')$, we thus have

$$\dot{\varepsilon}_{s,ref} = (c/\beta) \frac{\dot{\sigma}'}{\sigma'_{ref}} \left(\frac{\sigma'_{ref}}{\sigma'(0, \dot{\sigma}')} \right)^{\frac{1}{\beta}} \tag{7}$$

for the reference strain rate. Its value may now be fixed by selecting $\dot{\sigma}' = \dot{\sigma}'_{ref}$ for the reference loading rate and the corresponding intercept $\sigma'(0, \dot{\sigma}'_{ref}) = \sigma'_{ref}$ for the reference stress (cf. Fig. 1a). This is the choice that motivates the use of eq. (12) in [1], yielding the value

$$\dot{\varepsilon}_{s,ref} = (c/\beta) \frac{\dot{\sigma}'_{ref}}{\sigma'_{ref}}, \tag{8}$$

in which both $\dot{\sigma}'_{ref}$ and σ'_{ref} are now identified as experimentally determined quantities. We can then use the value of $\dot{\varepsilon}_{s,ref}$, so determined, to predict the value of $\sigma'(0, \dot{\sigma}')$ of the intercept for any other loading rate $\dot{\sigma}'$ - after substitution of (8) in (6) - from

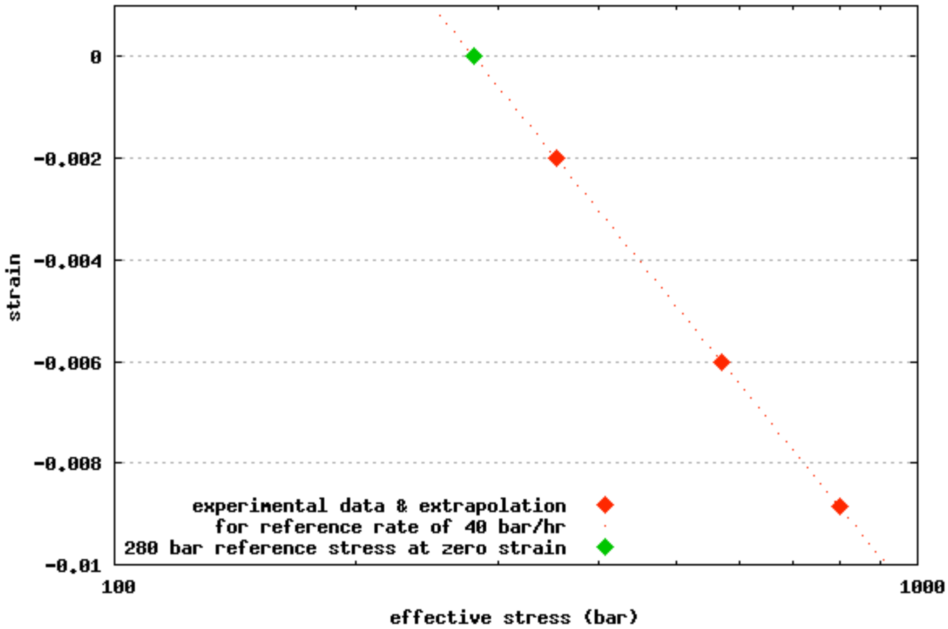
$$\sigma'(0, \dot{\sigma}') = \sigma'_{ref} \left(\frac{\dot{\sigma}}{\dot{\sigma}'_{ref}} \right)^{\beta} . \tag{9}$$

We note further that for any given strain $\varepsilon_s \neq 0$ the relationship

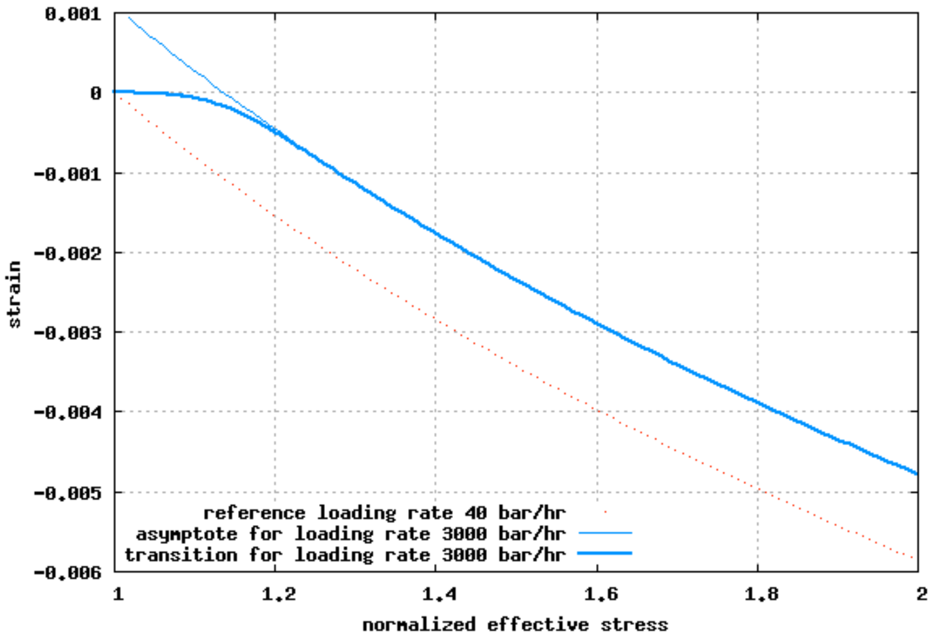
$$\sigma'(\varepsilon_s, \dot{\sigma}') = \sigma'(\varepsilon_s, \dot{\sigma}'_{ref}) \left(\frac{\dot{\sigma}'}{\dot{\sigma}'_{ref}} \right)^{\beta} . \tag{10}$$

also follows immediately from (5). For any two distinct loading rates $\dot{\sigma}'_1$ and $\dot{\sigma}'_2$ the stresses along the corresponding asymptotes at any given secular strain are thus related by

$$\sigma'_1(\varepsilon_s, \dot{\sigma}'_1) = \sigma'_2(\varepsilon_s, \dot{\sigma}'_2) \left(\frac{\dot{\sigma}'_1}{\dot{\sigma}'_2} \right)^{\beta} . \tag{11}$$



(a)



(b)

Figure 1. (a) Interpretation of data from constant loading rate experiment in terms of soft soil isotach model: Semi-log plot of and extrapolation of data to find reference stress $\sigma'_{ref} = \sigma'(0, \dot{\sigma}'_{ref}) = 280\,\text{bar}$; (b) transient solution (4) for secular strain ε_s as a function of σ'/σ'_{ref} between asymptotic solutions (5) for $\dot{\sigma}'_{ref} = 40\,\text{bar/h}$ and $\dot{\sigma}' = 3000\,\text{bar/h}$, respectively.

The last result turns out to be identical with a relationship proposed by de Waal [2] and employed by Pruiksma *et al.* as an essential building stone in their "isotach formulation" of the Rate Type Compaction Model (RTCM), which we discuss further below.

Although the 'secular' strain response to a constant loading rate may be better explained in the above way than by the incomplete[2] description in [1], the soft soil isotach model retains a fundamental flaw that will be appreciated easily by focussing on its prediction (3) of creep. We note first that differentiating and evaluating (3) at time t_0 yields

$$\dot{\varepsilon}_{s,0} = \dot{\varepsilon}_s(t = t_0) = \dot{\varepsilon}_{s,ref} \left(\frac{\sigma'_0}{\sigma'_{ref}} \right)^{b/c} , \tag{12}$$

which is consistent with (1) and the initial condition $\varepsilon_{s,0} = 0$. But (12) also relates the parameter $\dot{\varepsilon}_{s,ref}$ to the applied stress σ'_0, to the observed strain rate $\dot{\varepsilon}_{s,0}$, and to the reference stress σ'_{ref}. Indeed, suppose we took the latter equal to the applied stress, which seems a reasonable choice. This would imply $\dot{\varepsilon}_{s,ref} = \dot{\varepsilon}_{s,0}$, initself obviously 'reasonable', but equation (3) will predict a creep strain for this case, given by

$$\varepsilon_s = c \ln \left[1 + (\dot{\varepsilon}_{s,ref}/c)(t - t_0) \right] , \tag{13}$$

which is essentially independent of the applied stress. This behaviour is unlikely to have ever been observed in rocks. While it would be reasonable, in dealing with creep, to expect the existence of a threshold stress σ'_{ref} below which a given sample will exhibit negligible creep (for example the 3D state of stress applied to an undamaged sample that has been brought back to its *in-situ* state), the idea of creep proceeding essentially independent of any applied stress must be rejected. A way out of this dilemma must therefore lie in a modification of the basic constitutive assumption, avoiding wherever possible so-called *state parameters* and introducing genuine *material parameters*. This is likely to lead one in the direction of the SLS model of Section 5, but such an undertaking obviously lies beyond the scope of the present evaluation. - In summary we evaluate this model as follows:

- The model is inspired by experimental observations on soft soils, such as clay and peat, and efforts to model the uniaxial deformation response of these materials. Its relevance to the response of hard porous rocks to changes in pore pressure and stress. at temperatures near 100° C is therefore unclear from the outset.

[2]For example: In describing the input to the model simulation of test E004, as shown in their Fig. 2, the authors state "The parameter $\dot{\varepsilon}_{s,ref}$ is chosen such that equation (12) [i.e., our eq. 8] is valid" without explaining their choice and giving the impression that $\dot{\varepsilon}_{s,ref}$ could have been chosen independent of $\sigma'(0, \dot{\sigma}')$ and $\dot{\sigma}'$.

- The formulation of the soft soil isotach model involves the introduction of certain "state parameters", in addition to material parameters. The absence of a concrete example that would illustrate and motivate the need and precise nature of some of these parameters casts considerable doubt on the structure of the model, i.e., the rate-sensitive term, and the significance of the 'fits' of experimental data obtained with model predictions. In the above we have therefore dealt with this question in some detail by providing an interpretation of a hypothetical set of experimental data in terms of parameters of the soft soil isotach model.
- There remains, however, the following fundamental problem. A theory of rock deformation in response to pore-pressure and stress changes should always include a reference or *in situ* 'initial' stress state[3] at which the fluid-saturated reservoir rock will be in a state of thermodynamic equilibrium. Such an equilibrium state would imply fixed values of all pertinent 'internal variables', i.e., variables the would need to change in order to produce a macroscopic strain. These internal variables could be changed only by increases in their conjugate driving forces, brought about by changes in the macroscopic 'forces' such as stress, pore pressure, or temperature. This picture lies at the base of the concepts of a 'preconsolidation pressure' or of an 'overconsolidation ratio', well known from soil mechanics. The reference stress σ'_{ref} of the soft soil isotach model will in fact play the role of such an equilibrium stress, i.e., applying the stress $\sigma' = \sigma'_{ref}$ at any given time will cause the first term - the stress term - in equation (1) to vanish. It is however a characteristic of this model that the second term predicts a stress-free creep. This is an idea that appears without any physical basis in the present (isothermal and isochemical) context and it should therefore be rejected.
- In consequence of this defect of theory alone, we cannot recommend the use of the soft soild isotach of model. Moreover, we consider the purported agreement between theory and experiment in [1] as insufficiently documented and analyzed in the light of the question raised in the above. Especially when the obtained fits were constrained between limits known in advance (e.g., a single pair of isotachs), any deviations of the transient responses had almost necessarily to fall below the resolution of a graph and should therefore be considered inconclusive. The asymptotic 'isotachs' were therefore all that mattered in comparisons with experimental results, i.e., the alternative of a logarithmic or linear dependence of the strain on stress.

[3] In addition, of course, to an *in situ* temperature, pore pressure, and pore fluid composition.

2. Discussion of Stress-linearized Isotach Model (Section 4)

In this section the authors propose a linearized variant of the preceding soft soil isotach model, which is more in line with the experimental results of de Waal [2]. This involves essentially replacing the term $\ln \sigma'/\sigma_{ref}$ in (1) by a linear term to obtain the following "isotach equation"

$$\varepsilon_s = b\left(\frac{\sigma'}{\sigma_{ref}} - 1\right) - c \ln \frac{\dot{\varepsilon}_s}{\dot{\varepsilon}_{s,ref}} \tag{14}$$

for the secular strain. The equation is again readily integrated to give, with $\varepsilon_{s,0} = \varepsilon_s(t_0)$,

$$\varepsilon_s = \varepsilon_{s,0} + c \ln\left\{1 + e^{-\varepsilon_{s,0}/c}\,\frac{\dot{\varepsilon}_{s,ref}}{c} \int_{t_0}^{t} e^{(b/c)[\sigma'(t')/\sigma_{ref} - 1]}\,\mathrm{d}t'\right\} \tag{15}$$

so that one only needs to perform a simple quadrature for any given loading history $\sigma'(t)$.

If the loading rate $\dot{\sigma}'$ remains constant within a time interval $t_0 \leq t' \leq t$, an integration of (15) over stress, with $\mathrm{d}\sigma' = \mathrm{d}t/\dot{\sigma}'$, gives

$$\varepsilon_s = \varepsilon_{s,0} + c \ln\left\{1 + e^{-\varepsilon_{s,0}/c}\frac{\dot{\varepsilon}_{s,ref}}{b}\frac{\sigma'_{ref}}{\dot{\sigma}'}\left[e^{(b/c)(\sigma'/\sigma_{ref}-1)} - e^{(b/c)(\sigma'_0/\sigma_{ref}-1)}\right]\right\}. \tag{16}$$

With increasing σ' this solution approaches asymptotically the linear isotach

$$\varepsilon_s = b(\sigma'/\sigma_{ref} - 1) - c \ln\left(\frac{b}{\dot{\varepsilon}_{s,ref}}\frac{\dot{\sigma}'}{\sigma'_{ref}}\right). \tag{17}$$

The difference in stress, at a fixed value of ε_s, between two straight-lines asymptotes for loading rates $\dot{\sigma}'_1$ and $\dot{\sigma}'_2$ is now given by

$$\sigma'(\varepsilon_s, \dot{\sigma}'_1) - \sigma'(\varepsilon_s, \dot{\sigma}'_2) = \sigma_{ref} \ln\left(\frac{\dot{\sigma}'_1}{\dot{\sigma}'_2}\right)^{c/b}. \tag{18}$$

In contrast with (11), this difference remains constant with increasing strain (or stress).

In the same way as in the foregoing section, we may now determine the value of the reference strain rate $\dot{\varepsilon}_{s,ref}$ by plotting the strain ε_s versus the stress σ' of an actual data set - obtained for the constant loading rate $\dot{\sigma}'_{ref}$ - and by determining the intercept $\sigma'(0, \dot{\sigma}'_{ref})$ for $\varepsilon_s = 0$ of an extrapolated linear fit of the data. Putting $\varepsilon_s = 0$ in (17), we obtain

$$\sigma'_{ref} = \sigma'(0, \dot{\sigma}')\left[1 + \frac{c}{b}\ln\left(\frac{b}{\dot{\varepsilon}_{s,ref}}\frac{\dot{\sigma}'_{ref}}{\sigma'_{ref}}\right)\right]^{-1} \tag{19}$$

and if we equate σ'_{ref} to $\sigma'(0, \dot{\sigma}')$, we find that the reference strain rate is now given by

$$\dot{\varepsilon}_{s,ref} = b\frac{\dot{\sigma}'_{ref}}{\sigma'_{ref}}. \tag{20}$$

in terms of three experimentally determined quantities.

For creep at a constant applied load σ_0 equation (15) predicts the strain

$$\varepsilon_s = \varepsilon_{s,0} + b(\frac{\sigma'_0}{\sigma'_{ref}} - 1) + c \ln \left\{ e^{-(b/c)(\sigma'_0/\sigma'_{ref}-1)} + e^{-\varepsilon_{s,0}/c} \frac{\dot{\varepsilon}_{s,ref}}{c}(t - t_0) \right\}, \qquad (21)$$

where we are free to count the strain from $\varepsilon_{s,0} = 0$.

We may summarize this evaluation of the stress-linearized model as follows:

- The model is inspired by experimental observations on soft soils, such as clay and peat, and efforts to model the uniaxial deformation response of these materials. Its relevance to the response of hard porous rocks to changes in pore pressure and stress. at temperatures near 100° C is therefore unclear from the outset.
- Throughout their report, Pruiksma *et al.* unfortunately show very little interest in a comparison of creep data with model predictions. The report contains in fact only one such comparison (in Fig. 15), without giving the necessary detail on the theoretical relationships employed and material or 'model parameters' selected. It is therefore important to point out again the peculiar feature of 'stress-free creep', which reappears in (21) in the same way as had been noticed with equation (3). It would imply indeed that if one applied the load $\sigma'_0 = \sigma'_{ref}$, then even in an initial state of zero strain there would be a nonvanishing strain rate $\dot{\varepsilon}_s(t_0) = \dot{\varepsilon}_{s,ref}$ from $t = t_0$ to infinity! This prediction derives from the peculiar form of the constitutive equation (1) and its linearized version (14) and the poorly understood status of the parameters σ'_{ref} and $\dot{\varepsilon}_{s,ref}$ as "state parameters" [1]. Indeed, the authors appear to suggest here that the initial state of the Groningen reservoir rock, for example, is what thermodynamicists might call a 'stationary non-equilibrium state', characterized by continual dissipation (entropy production) upheld by some kind of geological agency other than stress and different from the (equally problematic) "geological loading rate" of the RTCM (see below).
- A model that predicts unphysical behaviour in creep cannot be regarded as a sound description of rate-dependent behaviour in general; it should therefore be rejected.

3. Discussion of Standard Linear Solid (SLS) Model (Section 5)

In this linear rheological model a 'Kelvin' (or 'Voigt') element is put in series with a 'Hooke' element (a spring); alternatively, a SLS model may be constructed by putting an elasto-viscous 'Maxwell' element (a spring and dashpot in series) in parallel with a Hookean spring. The latter is the more common arrangement [3], but the choice is immaterial in the 1D case under consideration; we shall therefore follow the authors, who opted for the first. The following discussion of the SLS model can remain brief, because the general theory of linear viscoelastic solids is well developed, has long been applied to geotechnical problems of various kinds in appropriate poro-viscoelastic versions[4], and has also been extended into the nonlinear elastic and viscoplastic range[5]. We shall therefore restrict our attention to the aspect of compaction delay as predicted by the linear SLS model.

The one-dimensional response of a standard linear solid may be taken to be governed by the equation

$$c_{m,a}\dot{\sigma}'\tau + (c_{m,a} + c_{m,b})(\sigma' - \sigma'_{ref}) = \varepsilon + \dot{\varepsilon}\tau \tag{22}$$

in which $\tau = c_{m,b}\,\eta$ is the relaxation time associated with the spring (of compliance $c_{m,b}$) and dashpot (of viscosity η) of the Kelvin element. Equation (22) differs from eq. (21) in [1] in that it involves the total strain ε rather than the 'secular' strain $\varepsilon_s = \varepsilon - \varepsilon_d$, ε_d being the 'direct' (elastic) strain.

Consider the solution of (22) for a constant loading rate $\dot{\sigma}'$, starting from $\sigma' = \sigma'_{ref}$ at $t = 0$, when $\varepsilon(0) = 0$. Putting $\sigma' - \sigma'_{ref} = \dot{\sigma}'t$ and $c_m = c_{m,a} + c_{m,b}$, we find that (22) can be brought into the form

$$\tau\frac{\mathrm{d}}{\mathrm{d}t}\big(\varepsilon - c_m\dot{\sigma}'t\big) + \varepsilon - c_m\dot{\sigma}'t = -c_{m,b}\dot{\sigma}'\tau. \tag{23}$$

The solution to this equation for $\varepsilon - c_m\dot{\sigma}'t = 0$ at $t = 0$ is given by

$$\varepsilon = [\,c_m - c_{m,b}\frac{1 - e^{-t/\tau}}{t/\tau}]\,\dot{\sigma}'t \tag{24}$$

and is plotted in red in Figure 2 for the parameter values listed there. The dotted asymptote to this transient solution of the constant-loading rate problem satisfies the linear

[4] An early step in this direction was taken by Tan Tjong Kie [4] in his impressive TH Delft doctoral dissertation that is still worth reading.

[5] See, for example, the rigorous study of Heidug [5], who presented an example of compaction delay in a material of the fading-memory type. Heidug has also emphasized the dependence of compaction strains in these materials on 3D states of stress.

relation

$$\varepsilon = c_m \dot{\sigma}' t - c_{m,b} \dot{\sigma}' \tau. \tag{25}$$

This corresponds to equation (24) in [1].

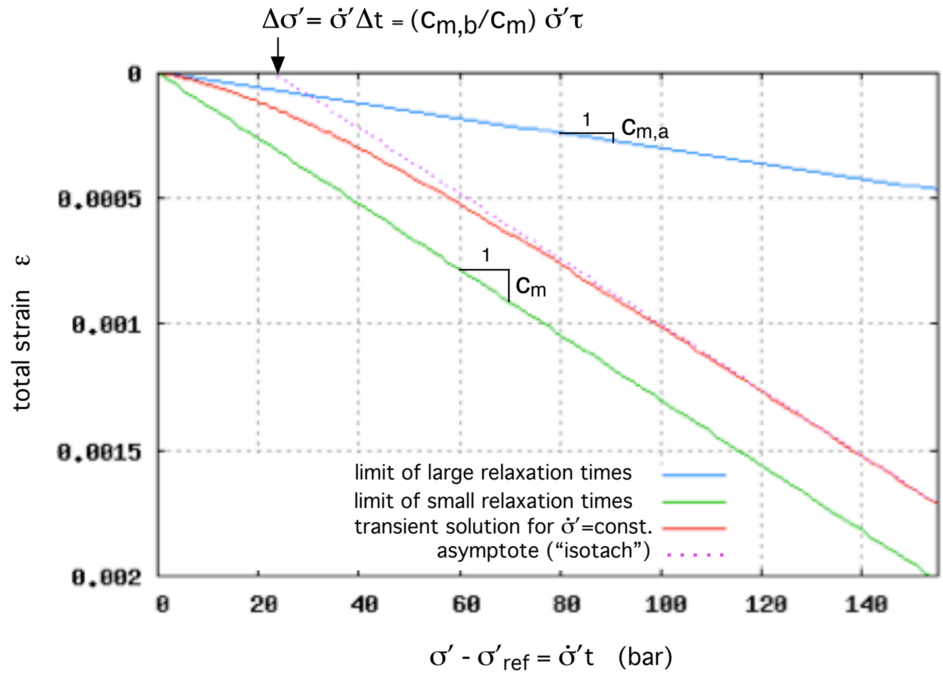


Figure 2. Example of constant-loading rate solution (24) for SLS Model, showing transient response for $\tau = 5\,\mathrm{yrs}$ and its (dotted) linear asymptote (isotach), assuming the values $c_{m,a} = 3 \times 10^{-6}\,\mathrm{bar}^{-1}$, $c_{m,b} = 10^{-5}\,\mathrm{bar}^{-1}$, and $\dot{\sigma}' = 6\,\mathrm{bar/yr}$. Also shown are the results for very small (green line) and very large (blue line) relaxation times τ (viz. viscosities η).

We also note here that (24) can be written

$$\varepsilon = c_{m,t}(t)\,(\sigma' - \sigma_{ref}) \tag{26}$$

in terms of a time-dependent compressibility

$$c_{m,t}(t) = c_m - c_{m,b} \frac{1 - e^{-t/\tau}}{t/\tau}, \tag{27}$$

which attains the limits $c_{m,t}|_{t \to 0} = c_m - c_{m,b} = c_{m.a}$ and $c_{m,t}|_{t \to \infty} = c_m$. This reveals an intrinsic time dependence of the material via a characteristic time τ, which is essentially independent of a loading rate; which of course is not to say that the loading rate will not enter into the solution of a typical initial value problem for the strain, as may be seen directly from (24).

What is learned here from this linear viscoelastic model is therefore that the observation of a loading rate dependence of laboratory results is interpreted in the simplest way in terms of an intrinsic relaxation phenomenon. A tentative interpretation of this kind will also immediately suggest various well-known techniques of determining relaxation times experimentally.

It is important to remember here that experiments designed to test mechanical behaviour of materials will, as a rule, be based on the assumption that the tested material displays the same behaviour in the field and in the laboratory. This says that an elastic modulus or a relaxation time determined on a rock sample in the laboratory will have the same value in the field. It is easy to see and well-known that this 'principle of material uniformity' will often violated, e.g., by the effects of core damage or the inability to determine and restore an *in-situ* state of stress, temperature, and pore fluid saturation or composition. In the present context and its emphasis on rate effects, this principle or assumption is therefore of particular significance. We shall illustrate this presently with a concrete attempt to interpret an experimental data set.

At a fixed loading rate $\dot{\sigma}'$ the intercept of the asymptote (25) at $\varepsilon = 0$ yields a stress difference

$$\Delta\sigma' = \dot{\sigma}'(c_{m,b}/c_m)\tau \tag{28}$$

equal to shift of the asymptote (25) from the fully relaxed solution (the green line in Fig. 2). It follows that the characteristic time of the SLS solid is determined by and may thus be obtained from

$$\tau = (c_m/c_{m,b})\,\Delta\sigma'/\dot{\sigma}'\,. \tag{29}$$

Since τ is a constant *material parameter* in the SLS model, as are the compressibilities c_m and $c_{m,b}$, the time interval or 'delay period'

$$\Delta t = \Delta\sigma'/\dot{\sigma}' = (c_{m,b}/c_m)\tau \tag{30}$$

is seen to be independent of the loading rate. As an application of the foregoing, let us consider a data set (made available by D. Doornhof of NAM Assen) for a uniaxial strain experiment performed on a core sample from well Oldorp 1-1. Figure 3 shows axial strain plotted versus axial effective stress ranging from 10 to 544 bar. The sample had been loaded first at a rate of 62 bar/h up to 323 bar, this was followed up by creep at 323 bar

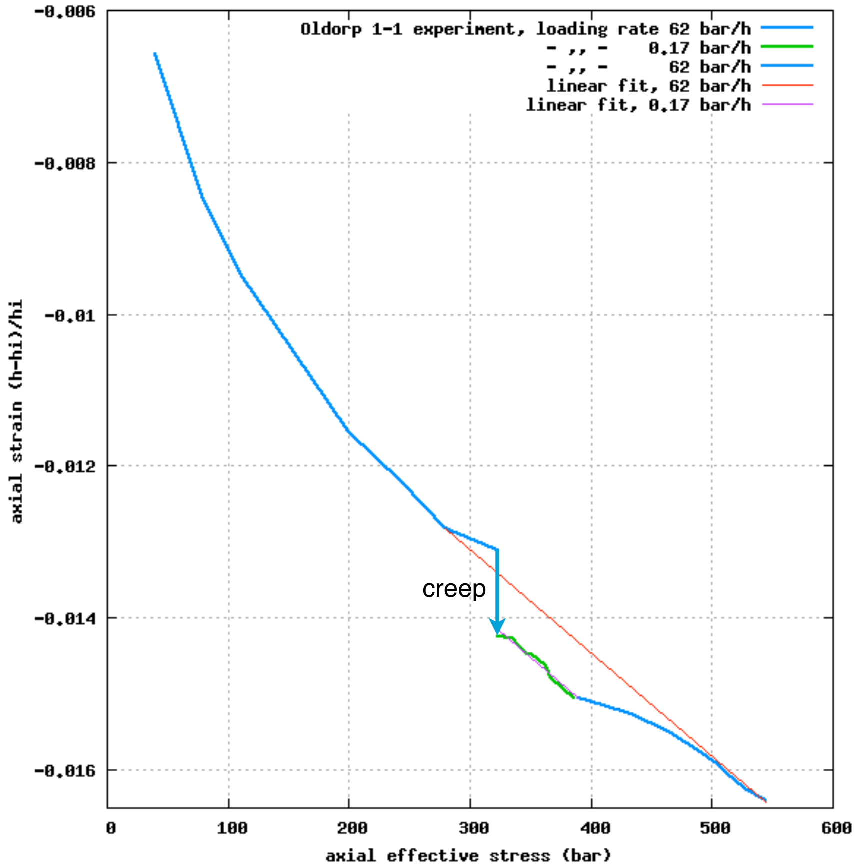


Figure 3. Uniaxial compaction data for Oldorp 1-1 core (provided by D. Doornhof, NAM Assen). Linear fits of 0.17 bar/h (green/magenta) and 62 bar/h (blue/red) trends yield both $c_m \approx 1.26 \times 10^{-5}$ bar^{-1}. Last 62 bar/h (blue) interval is shown in magnification, and evaluated in Fig. 4.

Unfortunately, the (available) description of the experiment provides no information on an in-situ stress state, which presumable had to be restored. Next to an effective vertical stress, which in [1] is typically found from an effective overburden load, a major question to be answered is that of the prevailing *in-situ* 'horizontal', tectonic stress. It is easy to see that the axial deformation response of a rock that is subjected to axial loads far beyond their initial value must depend significantly on the initial (effective) confining pressure (see, e.g., Heap *et al.* [6]). In other words, unless one has succeeded in restoring a known *in-situ* stress or else has explored experimentally a number of alternative tectonic stress scenarios, there will remain legitimate doubts about the degree of 'realism' achieved by an experiment.

The experimental protocol for the Oldorp 1-1 test records a 'compressibility' of $1.5 \times 10^{-5}\,\text{bar}^{-1}$ at 300 bar of effective stress. This stress level presumably corresponds to an initial effective 'vertical' stress. Being left with some uncertainty here, we decided to select the subsequent instant at which the loading rate was raised again to 62 bar/h from an intervening low level of 0.17 bar/h for an 'reference state'[6]. This occurred at $\sigma'_{ref} = 386\,\text{bar}$ of effective axial load at the strain $\varepsilon_{ref} \approx 0.015$. It is from this point onward that the data exhibit a clear transient response and approach to what may be taken as an asymptote and straight-line isotach for the 62 bar/h rate. This interval is selected and shown in magnification in Figure 4. Also shown there are straight lines giving the initial and final slopes of the transient, corresponding to the compressibilities $c_{m,a}$ and c_m, respectively. The values found for these are $c_m = 1.26 \times 10^{-5}\,\text{bar}^{-1}$ and $c_{m,a} = 3.57 \times 10^{-6}\,\text{bar}^{-1}$. One may then read off the value $\Delta\sigma'(\varepsilon_{ref})$ for the stress difference between the intercept of the line $\varepsilon_{ref} = const.$ and the transient's asymptote and the reference stress of 386 bar (see Fig. 4). We find $\Delta\sigma'(\varepsilon_{ref}) = 53\,\text{bar}$ and have thus gathered all the information we need to obtain the characteristic time from (29); we obtain $\tau = [12.6/(12.6-3.57)] \times 53/62 = 1.2\,\text{h}$! This is obviously an extremely small time constant. The result that can nevertheless serve as a warning not to expect a single experimental result to be conclusive in the sense that one could rely on it for predicting the rate-dependent response of the sample material at much lower loading rates, say. The most probable reason for finding such a short characteristic time is to be found in the extremely high loading rate, combined with the fact that the time-dependent response of sandstones very likely involves an entire spectrum of relaxation times, the larger ones of which would never come into play with such high a loading rate. This is a well-known phenomenon that one has attempted to capture with much more complex rheological models that involved various arrangements of viscoelastic elements of the SLS-type and sometimes include frictional or other strength elements[7].

A different possibility to test the simple SLS model of this section against 'reality' might

[6]This is clearly not an equilibrium state. We shall nevertheless disregard the consequences of making this assumption for the values of the compressibilities that we infer here.

[7]An early attempt to devise such models for soft soils is to be found in the Delft doctoral thesis of Tan Tjong Kie [4]. See also the report by Heidug [5] for a deeper theoretical analysis based on an internal variable formalism.

be to try and match a creep experiment. We shall consider this possibility further below in discussing the creep response of the RTCM, but will add here the following observations on the creep response as predicted by the SLS model.

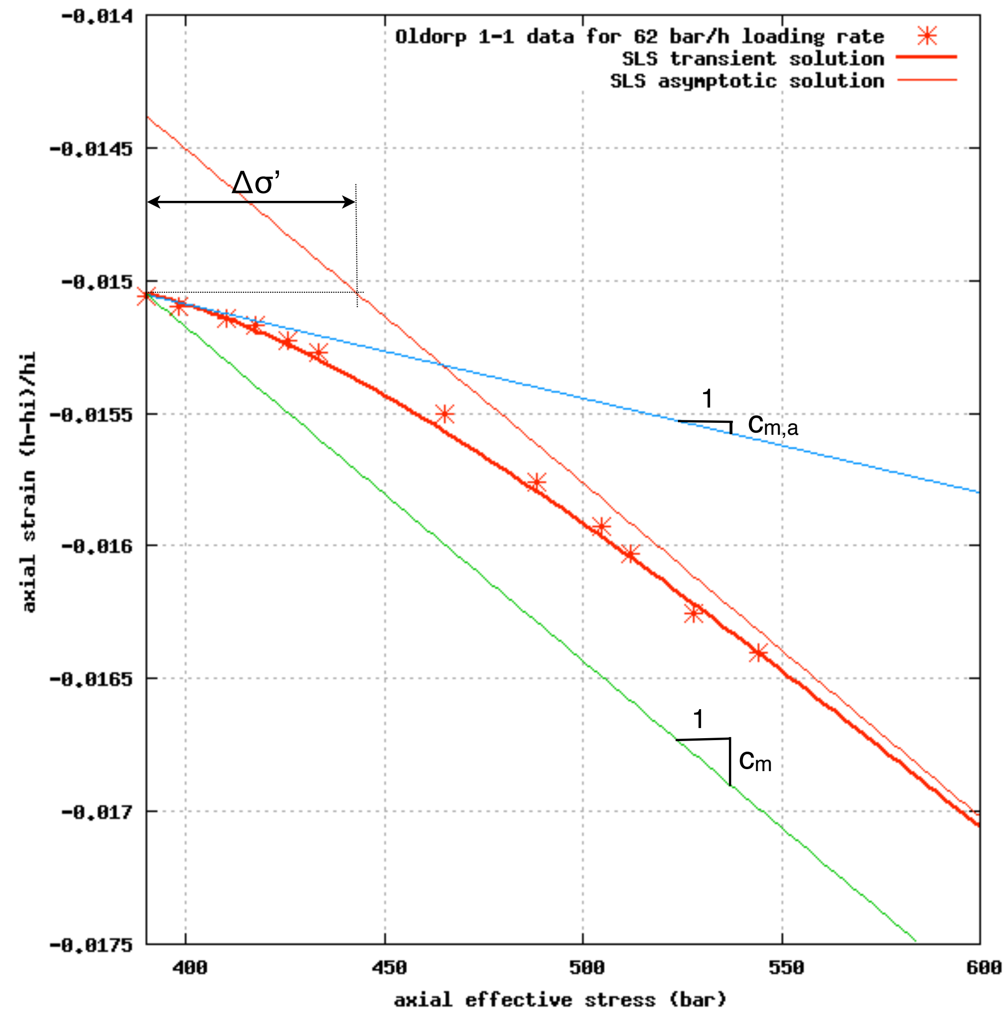


Figure 4. Interpretation of Oldorp 1-1 uniaxial strain data for loading rate $\dot{\sigma}' = 62\,\mathrm{bar/h}$ in terms of SLS model, yielding $c_m = 1.26 \times 10^{-5}\,\mathrm{bar}^{-1}$, $c_{m,a} = 3.57 \times 10^{-6}\,\mathrm{bar}^{-1}$, and $\Delta\sigma' = 53\,\mathrm{bar}$. Experimental data are those of the last (blue) loading interval in Fig. 3.

For the case of creep in response to a suddenly applied, constant 'excess load' $\sigma' - \sigma'_{ref}$ the solution of a correspondingly simplified equation (22) becomes

$$\varepsilon(t) = \left[c_{m,a} + c_{m,b}(1 - e^{-t/\tau})\right](\sigma' - \sigma'_{ref}) \tag{31}$$

This predicts an exponential decay of the creep rate toward some final maximum strain. In the usual experimental picture of the three stages of 'primary' (transient), 'secondary' (steady-state), and 'tertiary' (accelerating toward failure) creep, the 'exponential creep' predicted by the SLS model therefore corresponds to a form of primary creep. For a uniaxial loading experiment such a behaviour might indeed be expected, but creep experiments for consolidated sandstones, in particular, have in most cases been performed under different loading conditions, typically at fixed values of a differential stress. Under these conditions a prolonged period of secondary, steady-state creep is characteristically observed [7],[6],[8], whereby it serves notice that the term 'exponential creep' is used in this context to express the dependence of a steady creep rate on the applied differential stress, as distinct from a frequently observed 'power-law' dependence [7].

Note here the difference between relation (31) and the solutions (3) and (21) for creep as obtained for the soft soil model and its linearized version. The direct proportionality between stress and strain excludes any unphysical 'stress-free' creep. Material parameters are clearly identified as such, with prescriptions for their experimental determination that may be inferred directly from (31) or other appropriate solutions of (22). The definition of σ'_{ref} also poses no problem, as long as one agrees on the existence of a state of equilibrium at $\sigma' = \sigma'_{ref}$ and $\varepsilon = 0$ prior to some time t_{ref}, at which the stress is raised to a constant value $\sigma' > \sigma'_{ref}$. The general case of time-dependent loading $\sigma'(t)$ from such an equilibrium state, followed by creep from $t = t_0$ onward, is dealt with by matching the relevant solutions of (22), subject to appropriate continuity conditions for stress and strain at t_0. In Sections 5.3 and 5.4 of their report Pruiksma *et al.* discuss such more general solutions $\varepsilon(t)$ in the form of appropriate evolution integrals[8] of some time-varying load $\sigma'(t)$.

In summary we have the following comments:

- The SLS model expresses rate-dependent rock behaviour as a cause of subsidence delay in simple and transparent way. It represents a textbook example of a rheological model of a linear viscoelastic solid. As a comparison with experimental data for an Oldorp 1-1 core shows, models of this kind, which feature only a single relaxation time cannot capture the spectrum of responses encountered in practice with very different loading rates. This deficiency is therefore likely to vitiate any attempts to apply experimentally determined model parameters, such as a relaxation time, to

[8]These solutions could also have been obtained by elementary methods, i.e., without the use of Laplace transforms.

field conditions. Put in a simple way, rate-effects that are seen at extremely high loading rates in the laboratory will disappear under field conditions, while effects that appear in the field may never be observed in an experiment. Different experiments (e.g., involving the determination of relaxation spectra) and correspondingly more complex rheological models are needed to overcome this problem. Moreover, temperature remains a key variable in modeling viscoelastic as well as any other rate-dependent deformation behaviour. It cannot be ignored, in particular in extrapolating experimental results to field conditions.

- The primary creep response of the SLS model, in the form of an exponentially decaying strain, differs from the logarithmic dependence of a uniaxial creep strain on time that is typically observed with soft soils. Pruiksma *et al.* base their preference for this type of behaviour on the logarithmic creep predicted by the models discussed in sections 3, 4, and 6 of their report. Their only reference to logarithmic creep in the rock mechanics literature are the volumetric creep data obtained by Dewers and Hajash [9] in a series of experiments on sandstones, performed at 150 °C and 200 °C, respectively, in which intergranular pressure solution was identified as the main micro-mechanism of creep. At room temperatures, however, intergranular pressure solution may safely be excluded as a cause of measurable volumetric creep in sandstones. This choice of 'empirical support' is therefore invalid and betrays an unfortunate disregard for the physics of rate-dependent deformation. Indeed, Pruiksma *et al.* completely ignore other relevant sources of information with a greater bearing on the brittle creep behaviour of sandstones at room temperatures [7],[6] and at 75°C [8]. The following quotation from Heap *et al.* [8] speaks for itself:

> We show that an increase in temperature from 20 to 75°C significantly enhances stress corrosion cracking in all three sandstones, leading to (1) a systematic reduction in strength during constant strain rate experiments and (2) an increase by several orders of magnitude in brittle creep strain rates during stress-stepping creep experiments.

In conclusion, rheological models of rate-dependent deformation are unlikely to become useful, if insufficiently informed about relevant micro-mechanisms of deformation and relevant experimental work.

4. Discussion of Isotach formulation of the RTCM (Section 6)

Theoretical structure of the model

In this section the authors propose to formulate

> "... an isotach formulation of the RTCM that allows integration through a change in loading rate and describes creep in a consistent way, overcoming the limitations of the original RTCM presented by de Waal [2]."

The authors base their 1D formulation on the following three model assumptions:

1) A linear elastic 'direct strain'

$$\varepsilon_d = c_{m,a}(\sigma' - \sigma'_{ref}) \tag{32}$$

2) A linear stress-strain response at constant strain rates (i.e., an assumption of straight-line isotachs)

$$\varepsilon = c_m(\dot{\sigma}')\sigma' + \varepsilon_0 \tag{33}$$

3) A nonlinear relation between the coefficients $c_m(\dot{\sigma}'_1)$ and $c_m(\dot{\sigma}'_2)$ for any two different loading rates, given by

$$\frac{c_m(\dot{\sigma}'_1)}{c_m(\dot{\sigma}'_2)} = \left(\frac{\dot{\sigma}'_2}{\dot{\sigma}'_1}\right)^b \tag{34}$$

where $b \ll 1$ is small exponent. We note here that in de Waal's [2] original formulation the same relationship is assumed to hold for the net compressibilities $c_{m,b} = c_m - c_{m,a}$ at different loading rates, which determine the rate-dependent, 'secular' part $\varepsilon_s = \varepsilon - \varepsilon_d$ of the total strain, the elastic 'direct' strain having been disregarded by de Waal. Here de Waal's choice seems certainly more appropriate in view of the fact that relation (34) expresses a rate effect and should therefore not involve an elastic component of the total compressibility. Rather than attempting to correct this defect in the following, we shall therefore limit our discussion of the RTCM isotach formulation–when appropriate–to the 'secular' part of the total strain ε, without any distinguishing notation.

Because of (33), the relationship (34) also implies that at a given strain ε the stresses $\sigma'_1(\varepsilon, \dot{\sigma}'_1)$ and $\sigma'_2(\varepsilon, \dot{\sigma}'_2)$ along two different isotachs (with loading rates $\dot{\sigma}'_1$ and $\dot{\sigma}'_2$, respectively) will differ by

$$\sigma'_2(\varepsilon, \dot{\sigma}'_1) - \sigma'_1(\varepsilon, \dot{\sigma}'_2) = \sigma'_1(\varepsilon, \dot{\sigma}'_1)\left[\left(\frac{\dot{\sigma}'_2}{\dot{\sigma}'_1}\right)^b - 1\right]. \tag{35}$$

This is obviously identical with relation (11) found above for the stress ratio of the soft soil isotach model at a fixed secular strain ε_s.

Pruiksma *et al.* now proceed in their formulation to select a particular isotach as 'reference isotach', such that its zero-strain intercept on the $\sigma'-$axis coincides with a meaningful (or convenient) reference stress σ'_{ref}. According to (33) then

$$\varepsilon = 0 = c_{m,ref}\sigma'_{ref} + \varepsilon_0, \tag{36}$$

where $c_{m,ref}(\dot{\sigma}'_{ref})$ is the compressibility at the known loading rate $\dot{\sigma}'_{ref}$ that fixes the reference isotach. Thus, the selection of a reference state with known σ'_{ref} and $\dot{\sigma}'_{ref}$ is viewed by Pruiksma *et al.* as essential to the completion of the model. We quote the authors from p. 29 of their report:

> "The model has three material parameters: $c_{m,a}, c_{m,ref}, b$ and three state parameters $\dot{\sigma}'_{ref}$, σ'_{ref}, and ε_0. Setting these determines the behaviour of the model. Recall that $c_{m,ref}$ belonging to $\dot{\sigma}'_{ref}$ is arbitrary. It could be $c_{m,ref} = c_{m,lab}$ measured at $\dot{\sigma}'_{ref} = \dot{\sigma}'_{lab}$. However, here the reference isotach is chosen to be the one that crosses σ'_{ref} at $\varepsilon = 0$. This is the isotach that is currently in the field. The idea behind this is that in the field, before pressure depletion, the in situ vertical effective stress is σ'_{ref}, and this stress has been arrived at over geological time with a geological loading rate $\dot{\sigma}'_{ref}$. To this loading rate corresponds a slope $c_{m,ref}$. [Fixing ε_0 accordingly by (36) ...] This reduces the number of actual model input parameters to 5: $c_{m,a}, c_{m,ref}, b, \dot{\sigma}'_{ref}, \sigma'_{ref}$. $c_{m,ref}$ still needs to be determined. If $c_{m,lab}$ at $\dot{\sigma}'_{lab}$ is measured in the lab, the value of $c_{m,ref}$ follows from $c_{m,ref} = c_{m,lab}(\dot{\sigma}'_{lab}/\dot{\sigma}'_{ref})^b$ [i.e., from (34) above]."

Set up in this way, the RTCM isotach formulation can be illustrated as in Figure 5. Isotachs, obeying (33) & (34) and parametrized by the loading rate $\dot{\sigma}'$, emerge from the point $(0, \varepsilon_0)$ to cover an entire quadrant of the ε–σ' plane. This will allow one to determine the evolving strain ε for any given stress $\sigma'(t)$, either numerically as dicussed in [1], or else analytically as in the two special cases discussed further below–albeit with the restriction to secular strains except in the case of creep under a constant load. Shown in Figure 5 is such a solution for a case in which the loading rate is raised suddenly at the reference stress from $\dot{\sigma}'_{ref}$ to $\dot{\sigma}'_p$. Here the transient strain response traverses a fan of isotachs in approaching the isotach $\dot{\sigma}'_p = const.$ asymptotically. This could correspond to a real situation in the field, where $\dot{\sigma}'_p$ would correspond to a (constant) rate of decline of a reservoir pressure (i.e., increase in effective stress) and σ'_{ref} would be the *in-situ* initial stress, while $\dot{\sigma}'_{ref}$ would be the "geological loading rate" at which this state of stress had been reached.

However, we see no need to assume a "geological loading rate", and to infer from it[9] and from necessary lab data a value of the reference compressibility $c_{m,ref}$. We propose instead an alternative use of an RTCM isotach formulation, as summarized in Figure 5 and applied further below (cf. Fig. 11). This will still be based on observed linear data trends and an extrapolation to an expected depletion rate $\dot{\sigma}'_p$ in the field case for which one wishes to make a prediction. Before discussing an application of the proposed procedure, we must however consider first the type of predictions made by the RTCM in order to find out which data these will require. Accordingly, we shall discuss and apply closed form solutions (i) for the secular strain response to a constant loading rate, and (ii) for the full creep strain at a constant effective stress.

To obtain these solutions [1] Pruiksma *et al.* start, in effect, by differentiating (33) w.r. to time at a *fixed* loading rate, putting

$$\dot{\varepsilon} = c_m(\dot{\sigma})\dot{\sigma}' \,. \tag{37}$$

Substitution for $\dot{\sigma}'$ from (34), with $\dot{\sigma}'_1 = \dot{\sigma}'_{ref}$, then gives

$$\dot{\varepsilon} = c_m(\dot{\sigma})\,\dot{\sigma}'_{ref}\left(\frac{c_m(\dot{\sigma}')}{c_{m,ref}}\right)^{-1/b} .$$

By use of (33), the compressibility $c_m(\dot{\sigma}')$ is now replaced by the ratio $(\varepsilon - \varepsilon_0)/\sigma'$, to obtain

$$\dot{\varepsilon} = \left(\frac{\varepsilon - \varepsilon_0}{\sigma'}\right)\dot{\sigma}'_{ref}\left(\frac{\varepsilon - \varepsilon_0}{\sigma' c_{m,ref}}\right)^{-1/b} . \tag{38}$$

Note here the following difficulty in the above argument. According to Pruiksma *et al.*, equation (37) has to be seen as the result of differentiating (33) at a *fixed* loading rate, i.e., at a *fixed* value of the compressibility c_m. But in replacing the compressibility c_m in the subsequent steps by the ratio $(\varepsilon - \varepsilon_0)/\sigma'$, the authors obviously contemplate a general loading path $\sigma'(t)$ with continually varying compressibilities $c_m(\dot{\sigma}')$. As it stands, (37) only gives us the strain rate along the isotach for the loading rate $\dot{\sigma}'$. But the steps taken

[9] On a geological time scale a sedimentary layer will typically experience changes that will alter its constitution to an extent that will obviously invalidate the assumption of a uniform mechanical response throughout geologic time. The notion of a "geological loading rate" in the RTCM model thereby becomes meaningless and quite useless.

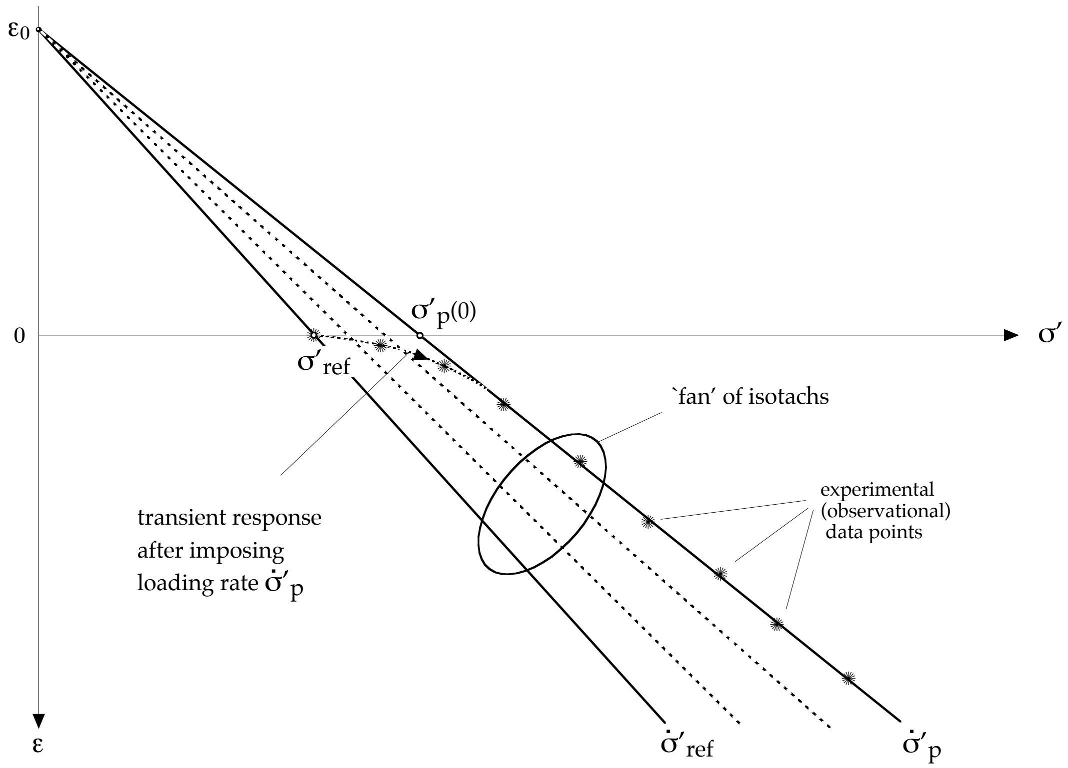


Figure 5. RTCM formulation: Isotachs with line parameter $\dot{\sigma}'$ converging toward fixed value of ε_0 satisfying (36). Given σ'_{ref} as *in situ* reservoir stress, finding ε_0 requires a reference compressibility $c_{m,ref}$, the latter being determined from eq. 34 in [1] (here eq. 34) in terms of laboratory values for c_m, $\dot{\sigma}'$, and a "geological loading rate" $\dot{\sigma}'_{ref}$. However, since the latter is rather meaningless in the present context, the following alternative implementation of an RTCM isotach formulation, based on (32)-(34), is suggested: Starting from a linear trend of ε vs. σ' data, which has been observed either in the field or in the laboratory for a given constant loading rate $\dot{\sigma}'_p$, one determines first its zero-strain and zero-stress intercepts $\sigma'_p(0)$ and ε_0, respectively. A second isotach, observed for a different loading rate, then suffices to determine the exponent 'b' and, by (32)-(34), the line parameter $\dot{\sigma}'$ for the isotach through any other zero-strain intercept $\sigma'(0)$. An extrapolation of laboratory observations to the lower loading rates, higher temperatures, and poorly constrained in situ confining pressures in the field is a step that will remain difficult to justifiy, however. A 'safer' application of the RTCM would be based on field data from observation wells or, with additional uncertainties, from subsidence data. The above procedure is applied further below to a given data set (cf. Fig. 11).

from (37) to (38) capture the change in the strain rate along a path that traverses a range of isotachs, including a path along which the load remains essentially constant, as in the case of creep. Time enters here, as does the loading rate, as a parameter. An integration of the strain rate (38) over time, for a given function $\sigma'(t)$, must therefore give us the strain $\varepsilon(t)$ accumulated up to time t. - Let us now examine two such solutions of (38) that are readily obtained[10].

(i) *Solution for a constant loading rate* $\dot{\sigma}'_p$

If the strain accumulates at a constant loading rate $\dot{\sigma}'_p$, with $\sigma' = \dot{\sigma}'_p(t - t_i) + \sigma'_i$, then $\mathrm{d}t = \mathrm{d}\sigma'/\dot{\sigma}'_p$ and (38) can, by use of (34), be brought into the form

$$\mathrm{d}(\varepsilon - \varepsilon_0)^{1/b} = c_{m,ref}^{1/b}\Big(\frac{\dot{\sigma}'_{ref}}{\dot{\sigma}'_p}\Big)\mathrm{d}\sigma'^{1/b} = c_m^{1/b}\mathrm{d}\sigma'^{1/b}$$

which is immediately integrated for $\varepsilon(t = t_i) = \varepsilon_i$ and $\sigma'(t = t_i) = \sigma'_i$, to give

$$\varepsilon(\sigma') = \varepsilon_0 + \big[(\varepsilon_i - \varepsilon_0)^{1/b} + c_m^{1/b}(\sigma'^{1/b} - \sigma'^{1/b}_i)\big]^b. \tag{39}$$

This solution (for the secular strain) evidently approaches the straight-line isotach (33) asymptotically. Note that apart from the initial state $(\sigma'_i, \varepsilon_i)$ and the exponent 'b', the solution involves only the slope of its asymptote, i.e., of the final isotach reached in traversing the 'fan of isotachs' of Figure 5. The trajectory of this transient solution in the $\sigma' - \varepsilon$ plane is therefore bounded between limits that are known in advance, leaving little space for 'unsatisfactory' curve fits!

(ii) *Solution for creep at a constant load* σ'_c

This solution is obtained by expressing equation (38) in terms of the variable $((\varepsilon - \varepsilon_0)/c_{m,ref}\sigma'_c)$ and integrating over time, with the result

$$\varepsilon = \varepsilon_0 + c_{m,ref}\sigma'_c\big\{(\varepsilon_c - \varepsilon_0)^{1/b}/(c_{m,ref}\sigma'_c)^{1/b} + \dot{\sigma}'_{ref}(t - t_c)/(b\sigma'_c)\big\}^b. \tag{40}$$

Here $\varepsilon_c = \varepsilon(t = t_c)$ equals a known initial strain, for example the strain accumulated during a preceding period of constant rate loading. The difficulty with this result is that the present isotach formulation of the RTCM suggests that it applies only to this rather specific experimental 'scenario'. One can see, the time is divided in (40) by a 'reference

[10]That closed-form solutions always provide the best understanding of a theory is seen also in the case of solution (40) for creep and its approximation (46), which refutes the criticism of Pruiksma *et al.* of an earlier result by de Waal (see below).

time' $\sigma_c/\dot{\sigma}'_{ref}$ which is *not* of the nature of a *material* property. Moreover, suppose one had chosen as a different experimental scenario in which a sample would first be brought to an equilibrium state (e.g., at the stress σ'_{ref}) and from there loaded rapidly up to the stress σ'_c, followed by the observation of creep. What should be the driving stress in this case and, again, which characteristic time would govern the evolution of the creep strain in this case? The RTCM has simply no answer to these questions. Moreover, in the example considered hereafter, creep turns out to be predicted just as well by a constant-loading rate solution of the RTCM.

We take example 8.4.3 of Pruiksma *et al.* to illustrate and discuss the above solutions, and also to compare them with the numerical results obtained in [1]. In this example loading commences at $\varepsilon_i = 0$ and the reference stress $\sigma'_{ref} = \sigma'_i = 300\,\text{bar}$ at the constant rate $\dot{\sigma}'_p = 5\,\text{bar/yr}$ up to $t_c = 60\,\text{yrs}$, when it is followed by creep at $\sigma'_c = 600\,\text{bar}$. Pruiksma *et al.* assume here that prior to production a "geological loading rate" $\dot{\sigma}'_{ref} = 3.2 \times 10^{-3}\,\text{bar/yr}$ has been in effect[11,12].

Figure 6 gives the solution (39) for the first 60 years, using the data listed there. This is to be compared with Fig. 14 in [1]. A closer look reveals an expected disagreement in the early, transient section between the present secular strain solution (39) and the numerical results of [1] for the total strain. This is of little consequence, however, since the asymptotic approach of the 5 bar/yr isotach is unaffected. We note again that - next to the exponent 'b' - only the compressibility c_m (or $c_{m,prod}$) enters into (39). Thus, while (39) requires a value for ε_0, the predicted transient *does not depend* on a "a geological loading rate", i.e., on the line parameter of the isotach through the point $(\sigma'_{ref}, \varepsilon = 0)$. Also, the compaction strain (which is always plotted along the negative ε-axis in this report) is found to become effectively delayed over a stress interval corresponding to the difference between the zero-strain intercepts of the isotachs (33) for the loading rates $\dot{\sigma}'_p$ and $\dot{\sigma}'_{ref}$, respectively. This is the stress interval given by (35), which by the above can be written

$$\begin{aligned} \Delta\sigma' = \sigma'_p(0) - \sigma'_{ref} &= (c_{m,ref}/c_{m,prod} - 1)\sigma'_{ref} \\ &= (1 - c_{m,prod}/c_{m,ref})\sigma'_p(0). \end{aligned} \tag{41}$$

[11] Further below we show how one can avoid the way in which Pruiksma *et al.* attempt to deduce a value for $\dot{\sigma}'_{ref}$ from a time constant τ that is defined in one of the other models discussed in their report.

[12] Note also that here and in several other places of the report, the figure legend (of Fig. 14) is unclear, but that - judging by our own results - the reference isotach agrees indeed with a loading rate of 3.2×10^{-3} rather than 10^{-5} bar/y.

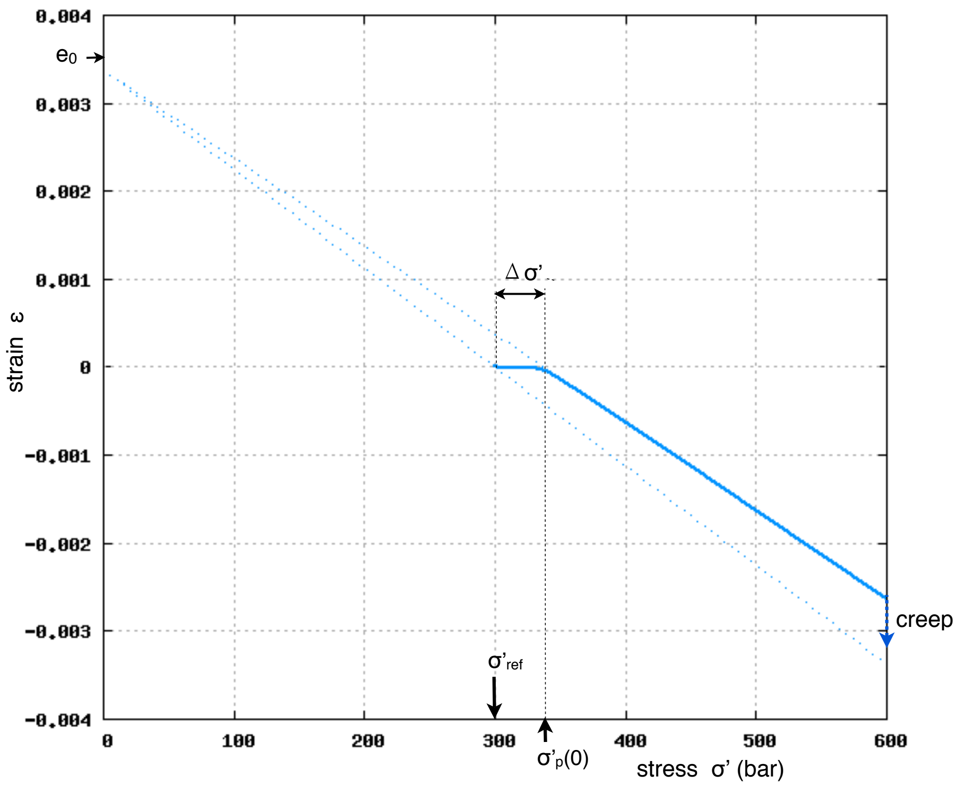


Figure 6. Compaction strain (taken negative) as predicted by RTCM solution (39) for constant loading rate $\dot{\sigma}'_p$ (example 8.4.3 in [1] recalculated), for parameter values consistent with (34): $c_{m,ref} = 1.12 \times 10^{-5}\,\text{bar}^{-1}$, $c_{m,prod}(\dot{\sigma}'_p) = 10^{-5}\,\text{bar}^{-1}$, and $b = 0.015$. Depletion starts from $\varepsilon_i = 0$ at $\sigma'_{ref} = \sigma'_i = 300\,\text{bar}$, by switching from a "geological loading rate" $\dot{\sigma}'_{ref} = 3.2 \times 10^{-3}\,\text{bar yr}^{-1}$ to $\dot{\sigma}'_p = 5\,\text{bar yr}^{-1}$. After 60 yrs of production, depletion is followed by 100 years of creep (cf. Fig. 9) at $\sigma'_c = 600\,\text{bar}$. Also shown is the dotted straight-line isotach (33) for the loading rate $\dot{\sigma}'_{ref}$ and $\dot{\sigma}'_p$, respectively, the latter - with with $\sigma'_p(0) = 336\,\text{bar}$ - being approached by (39) with a delay in compaction over the stress interval $\Delta\sigma'$ given by (41).

For the example of Figure 6 one finds $\Delta\sigma' = 36\,\text{bar}$. In Figure 7 this result (the green lines) may be compared with others for a lower and higher production rate, respectively.

If the loading rate $\dot{\sigma}'_p$ is assumed constant, relation (39) allows one to express the strain as a function of time in the following way

$$\varepsilon(t) = \varepsilon_0 + \left\{(\varepsilon_i - \varepsilon_0)^{1/b} + c_{m,prod}^{1/b}\left[\left(\dot{\sigma}'_p(t - t_i) + \sigma'_i\right)^{1/b} - \sigma'^{1/b}_i\right]\right\}^b. \tag{42}$$

At large times this is seen to approach the straight-line asymptote

$$\varepsilon(t) = \varepsilon'_i + c_{m,prod}\,\dot{\sigma}'_p(t - t_i). \tag{43}$$

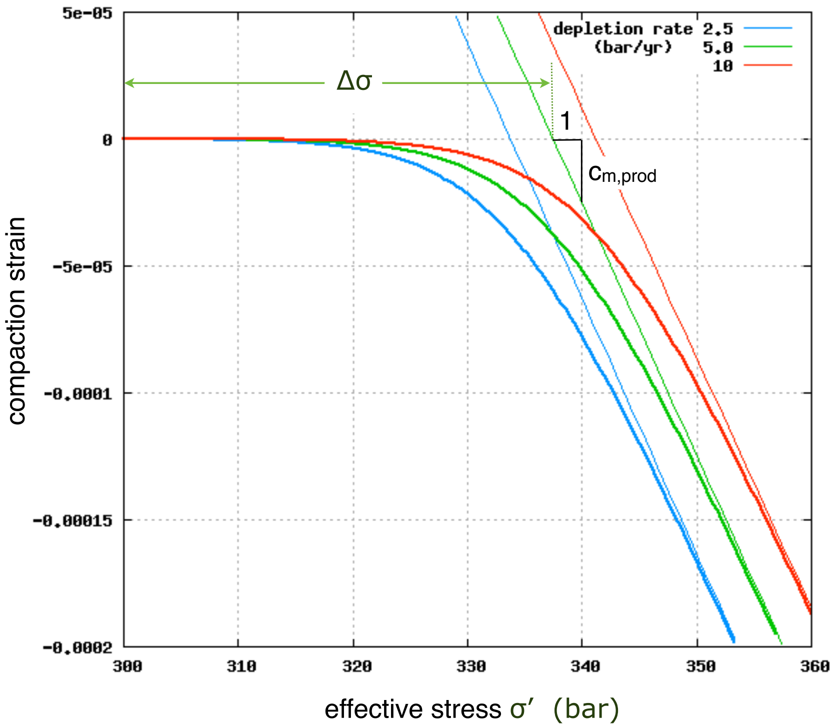


Figure 7. Detail of example of Fig. 6 for more than one loading rate, illustrating dependence of stress interval $\Delta\sigma'$ of delayed compaction on depletion rate $\dot{\sigma}'_p$ (eqs. 41 & 34); fixed values of $c_{m,ref}, \sigma'_{ref}, \dot{\sigma}'_{ref}$, and b are those of Fig. 6.

Here $\varepsilon'_i = \varepsilon_0 + c_{m,prod}\,\sigma'_i$ is the strain attained along the asymptote (33) at $\sigma'(t_i) = \sigma'_i$, which depends through $c_{m,prod}$ on the loading rate $\dot{\sigma}'_p$.

In Figure 8 we have ploted (42) and its asymptote (43) for three different depletion rates, taking $t_i = 0$. In comparison with Figure 7 the positions of the graphs for different loading rates are now reversed, reflecting a dominant effect of the depletion rate σ'_p.

Taking a closer look at the relationship between depletion rate and compaction delay, we note that (41) translates into the following expression for the 'delay period'

$$\Delta t = \Delta\sigma'/\dot{\sigma}'_p = \left[1 - c_{m,prod}/c_{m,ref}\right]\sigma'_p/\dot{\sigma}'_p, \tag{44}$$

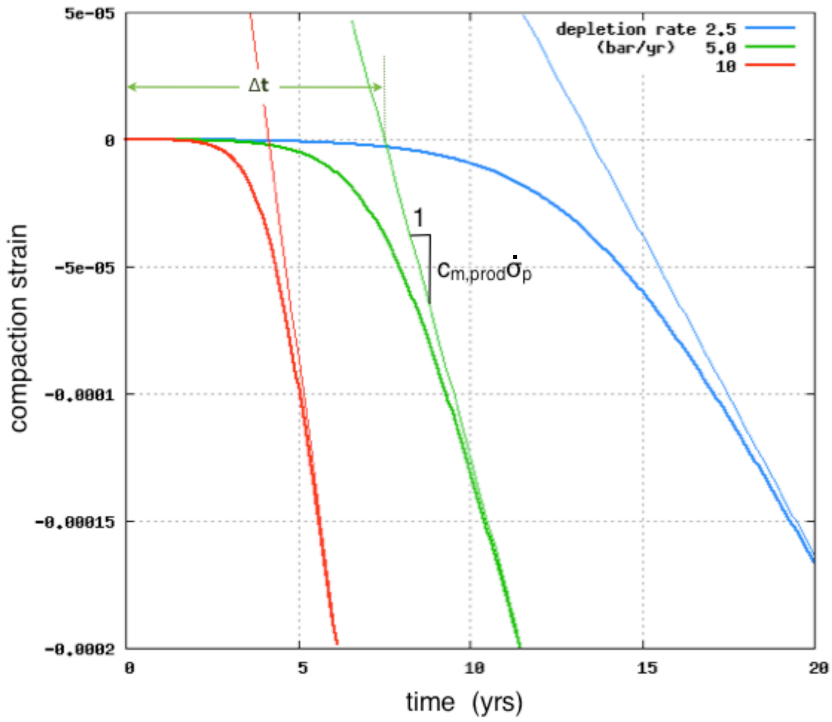


Figure 8. Dependence of delay period Δt on depletion rate $\dot{\sigma}'_p$ illustrated by plots of (42) & (43) for three different depletion rates, assuming fixed values for $c_{m,ref}, \sigma'_{ref}, \dot{\sigma}'_{ref}$, and b equal to those of Fig. 6. For $\dot{\sigma}'_p = 5\,\mathrm{bar/yr}$ (green lines) $\Delta t = 7.5\,\mathrm{yrs}$, as shown.

and this must be seen in contrast to the behaviour of the SLS solid, for which the delay period was found in to be essentially independent of the loading rate (cf. the remark following eq. 29). Expression (44) also highlights a characteristic feature of the RTCM, which is the absence of a genuine material parameter in control of rate dependent behaviour. In (44) it is the ratio $\sigma'_p/\dot{\sigma}'_p$ that appears in the role of a characteristic 'time function' rather than as a fixed characteristic time. Indeed, rewriting (35) for any isotach $\dot{\sigma}'_p$ and an (arbitrarily) fixed reference isotach $\dot{\sigma}'_{ref}$, one finds

$$\frac{\sigma'_p(\varepsilon)}{\dot{\sigma}'_p} = \left(\frac{\dot{\sigma}'_{ref}}{\dot{\sigma}'_p}\right)^{1-b} \frac{\sigma'_{ref}(\varepsilon)}{\dot{\sigma}'_{ref}},$$

which shows that the ratio $\sigma'_p(\varepsilon)/\dot{\sigma}'_p$ is a unique function of the ratio $\dot{\sigma}'_{ref}/\dot{\sigma}'_p$, independent of one's choice of reference isotach, as is immediately verified by applying this same relationship to any two potential reference isotachs. As with (34), there remains however the question of the range of validity of a universal scaling relationship of this type.

Next, let us consider to creep response, as predicted by solution (40) of the RTCM. This solution, for example 8.4.3 in [1], can now be joined together with solution (39) for a constant loading rate to exhibit the complete compaction history of example 8.4.3. This is shown Figure 9, which is found to agree well with the RTCM results of Fig. 15 in [1], thereby providing a check on the numerical scheme used there.[13]

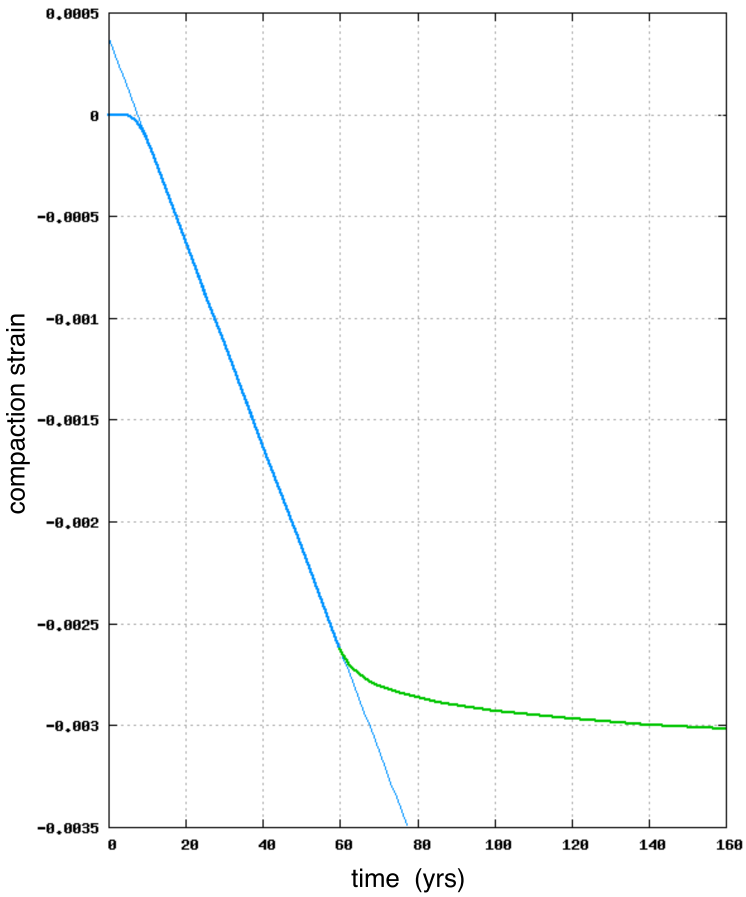


Figure 9. Compaction strain vs. time at constant depletion rate $\dot{\sigma}'_p = 5\,\text{bar/yr}$ (eq. 42, blue line), followed by creep at $\sigma'_c = 600\,\text{bar}$ (eq. 40, green line), for same data as in Fig. 6 (cf. example 8.4.3, Fig. 15 in [1]). Also shown is straight-line asymptote (isotach) (43).

[13]Note again that elastic strains remain constant during creep and will therefore not affect the shape of the creep curve.

We add here the following observations on solution (40) for the creep strain. First, we note that by use of (34) we have $(c_{m,ref}\,\sigma'_c)^{1/b} = (c_{m,prod}\,\sigma'_c)^{1/b}\dot{\sigma}'_p/\dot{\sigma}'_{ref}$ so that (40) can also be written

$$\varepsilon = \varepsilon_0 + c_{m,prod}\,\sigma'_c\big\{(\varepsilon_c - \varepsilon_0)^{1/b}/(c_{m,prod}\,\sigma'_c)^{1/b} + \dot{\sigma}'_p(t - t_c)/b\sigma'_c\big\}^b. \tag{45}$$

For values of $b \ll 1$ this can approximated by the logarithmic 'creep law'

$$\varepsilon - \varepsilon_c = bc_{m,prod}\,\sigma'_c \ln\big\{1 + \dot{\sigma}'_p(t - t_c)/b\sigma'_c\big\}.$$

If we expand the fraction under the logarithm by a *constant* compressibility c_m, we can write this

$$\varepsilon - \varepsilon_c = bc_{m,prod}\,\sigma'_c \ln\big\{1 + \frac{\dot{\varepsilon}_c}{bc_m\,\sigma'_c}(t - t_c)\big\}, \tag{46}$$

in terms of a strain rate $\dot{\varepsilon}_c = c_m\dot{\sigma}'_p$. Although we are free to choose the constant c_m, the particular choice $c_m = c_{m,prod}$ allows us to interpret $\dot{\varepsilon}_c$ as the strain rate at time $t = t_c$, i.e., at the beginning of the creep phase. The expression (46) then becomes identical with de Waal's result in [2] (cf. eq. 39 in [1]).

Pruiksma *et al.* have raised an objection against (46), arguing that c_m should not be treated as a constant, but should be allowed to vary as the strain crosses the 'fan' of isotachs in the course of creep (cf. the discussion following their eq. 39). We see that this particular objection is unwarranted, in that the integration carried out in the closed-form solutions (40) or (45) already takes care of the variation in compressibility.

In Figure 10 the creep interval of the strain history of Figure 9 is shown in more detail. This illustrates the accuracy to be expected for the logarithmic creep law (46) for small enough values of the exponent b. But perhaps somewhat surprising is the agreement of the (green) solution (40) for creep with the (blue) solution (42) for the case in which loading continues after $t = t_c$ at the reference loading rate $\dot{\sigma}'_{ref}$. This agreement will not persist indefinitely, however, since the blue line must approach its 'reference isotach' asymptotically, but it would certainly be accurate enough for 'practical purposes' up to large times.

All this cannot dispel a fundamental doubt about the physical reasonableness of the above 'creep laws'. Indeed, why should the creep response depend on a reference loading rate $\dot{\sigma}'_{ref}$ and compressibility $c_{m,ref}$ and, as demanded by (40)? The alternative dependence, as in (45), on the compressibility $c_{m,prod}$ and on the loading rate $\dot{\sigma}'_p$ just prior to the creep interval seems to make more sense at first, but the persistence of this effect in time with the ratio $\dot{\sigma}'_p(t-t_c)/\sigma'_c$ playing the role of a fictitious excess load is equally unlikely as is the persistent effect of an initial strain rate $\dot{\varepsilon}_c$ implied by the logarithmic approximation (46); both effects would in fact imply a perfect memory of the past.[14]

A further question one must ask is: Which quantity in (40) can possibly account for the temperature sensitivity of creep rates? The only possible candidate appears to be the exponent 'b', because the ratio $\sigma'_c/\dot{\sigma}'_p$ being imposed independent of temperature, 'b' remains the only parameter capable of controlling the time scale of creep by a dependence on temperature. However, little appears to be known about a temperature dependence of the exponent 'b'.

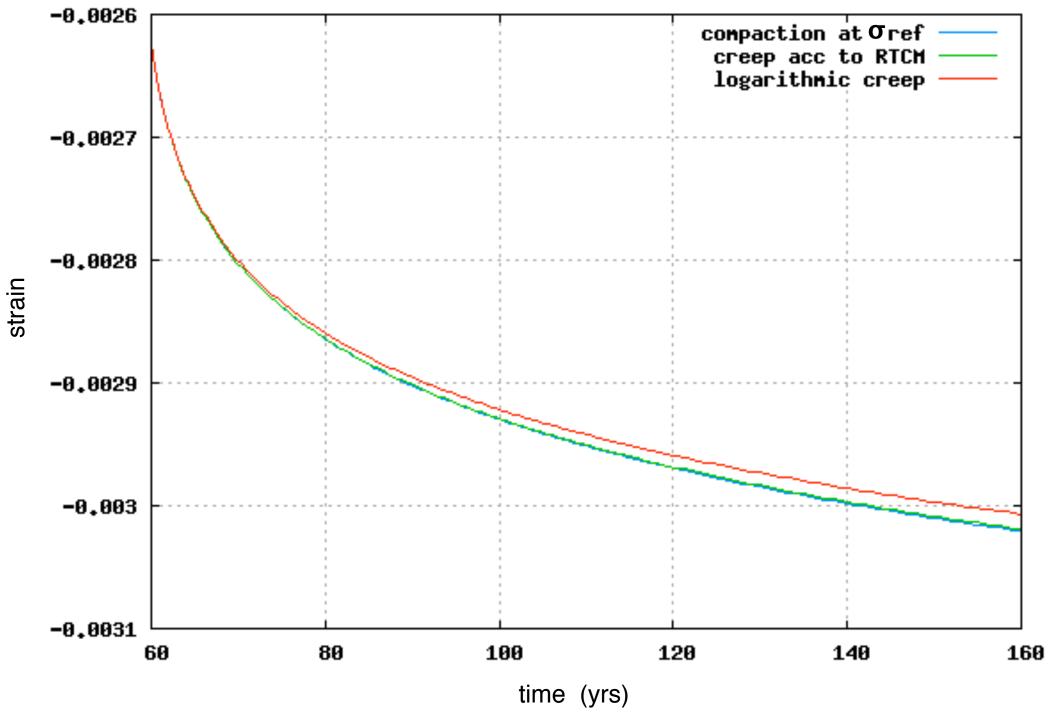


Figure 10. Detail for creep interval of Fig. 9: Compaction strain vs. time for constant depletion rate $\dot{\sigma}'_{ref} = 3.2 \times 10^{-3}$ bar/yr, starting at $t_i = t_c = 60$ yrs from $\sigma'_c = 600$ bar, according to eq. (42) (in blue); creep strain at $\sigma'_c = 600$ bar (i) according to exact result (40) (in green, same as in Fig. 9), and (ii) according to approximation (46) (in red).

[14]See also [10] for a critical analysis and discussion of the 'creep law' (46).

Application of the RTCM, based on an experimental data set

We turn now the an application of the RTCM to the Oldorp 1-1 experimental data set of Figure 3, which has already been interpreted in terms of the SLS model. Shown again in Figure 11 are the Oldorp 1-1 experimental data, with the 0.17 bar/h loading interval followed by the final 62 bar/h loading interval. The estimated straight-line asymptote of the latter has been fitted to the final stage of the first 62 bar/h loading interval to obtain a first isotach; this fixes the value $\varepsilon_0 = 0.00895$. A second isotach, fitting the (green) 0.17 bar/h experimental trend, is drawn to meet the first at the same zero-stress intercept ε_0. The point $(\sigma'_i = 386, \varepsilon_i = 0.015)$, at which the change from 0.17 bar/h to 62 bar/h occurred, defines the 'initial state' of the final 62 bar/h transient. The strain $\varepsilon = \varepsilon_i = 0.015$, rather than $\varepsilon = 0$, is used subsequently to define the intercepts of various straight-line isotachs.

(i) *Comparison with data obtained at a constant loading rate*

Making use of relations (33)-(35), the two interpreted isotachs now yield the following information: $\Delta\sigma'(\varepsilon_i) = 53\,\text{bar}$, $c_m(62) = 1.37 \times 10^{-5}\,\text{bar}^{-1}$, $c_m(0.17) = 1.578 \times 10^{-5}\,\text{bar}^{-1}$, and $b = 0.24$.[15] These suffice to obtain the field production isotach for an assumed loading rate $\dot{\sigma}'_p = 5\,\text{bar/y}$, as well as a prediction of the transient (heavy magenta) compaction strain from (39), starting at an assumed *in situ* stress $\sigma'_{ref} = 300\,\text{bar}$. The proposed procedure also yields the isotach through σ'_{ref} for a *fictitious* and meaningless geological loading rate of 0.0389 bar/y, which is not needed here, however. We note again the unrealistic stiff response predicted for the early stages of compaction in the field, which is due to our neglect of elastic strains. Taking the latter into account would obviously improve this prediction. Unfortunately, the modification of this part of de Waal's work by Pruiksma *et al.* does not seem to accomplish the required generalization in a proper way, as was already pointed out in the above, so that some work on this problem remains to be done.

The neglect of elastic strains appears in magnification in Figure 12, where we have attempted a match of the last, transient part of the experimental data. Again, the solution remains of course constrained by its asymptote (isotach) which is known in advance. The total 'compaction delay' $\Delta\sigma'$ is also known therefore, but its predicted evolution in time is more rapid than seen in the experiment.

[15]Note here that the estimate for $c_m(62)$ differs from the uniform value of c_m obtained for the parallel isotachs of the SLS model in Fig. 4.

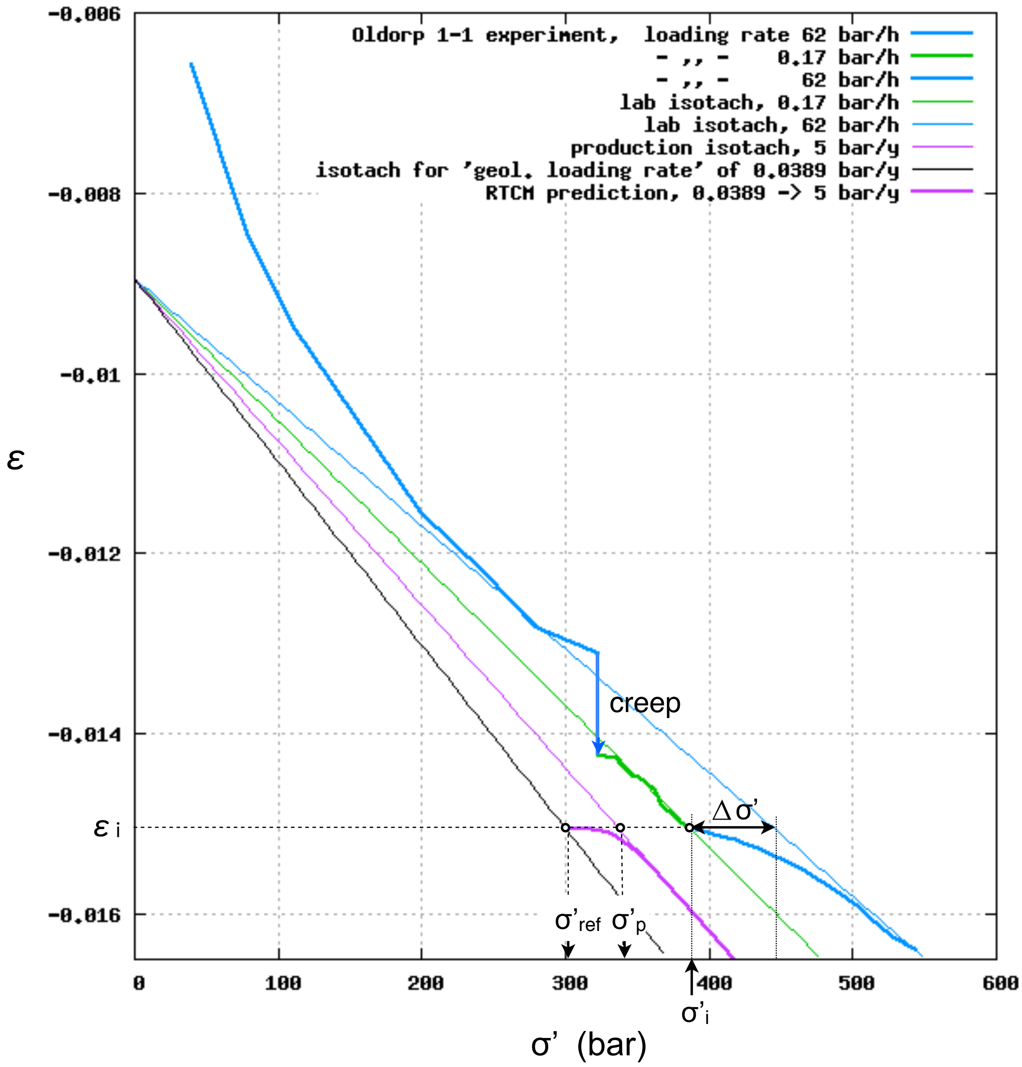


Figure 11. RTCM "field prediction" based on Oldorp 1-1 core data (heavy blue and green lines) for constant loading rate experiments and intervening creep interval at $\sigma_c = 323\,\text{bar}$. 'Initial state' ($\sigma'_i = 386, \varepsilon_i = 0.015$) of experimental transient fixes reference strain $\varepsilon_i \neq 0$. Fitted isotachs for loading rates 62 bar/h and 0.17 bar/h yield $c_m(62) = 1.37 \times 10^{-5}\,\text{bar}^{-1}$ and $c_m(0.17) = 1.58 \times 10^{-5}\,\text{bar}^{-1}$, respectively, and determine $\varepsilon_0 = 0.00895$ and $b = 0.024$. These suffice to obtain field production isotach for $\dot{\sigma}'_p = 5\,\text{bar/y}$, with compressibility $c_{m,prod}$, intercept $\sigma'_p(\varepsilon_i)$, and–from (39)–the field production transient (heavy magenta) starting at *in situ* stress $\sigma'_{ref} = 300\,\text{bar}$. Also shown, although not needed, is isotach through σ'_{ref} for *fictitious* "geological loading rate" of 0.0389 bar/y.

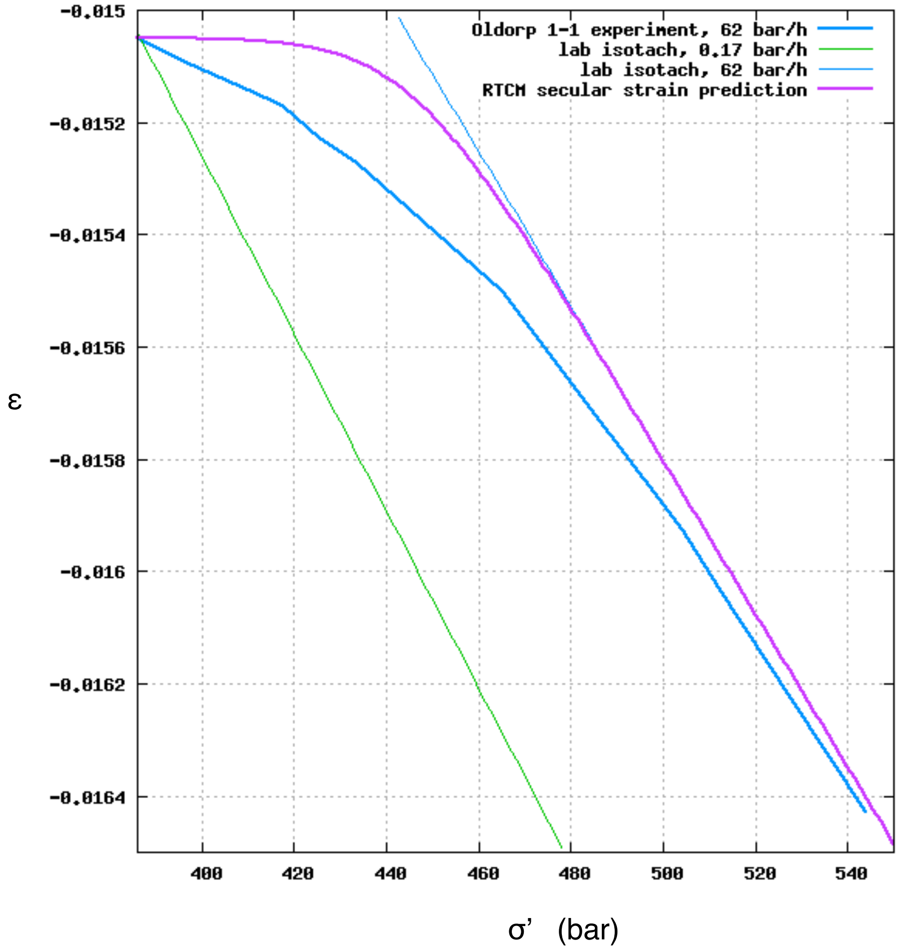


Figure 12. Oldorp 1-1 experimental data for 62 bar/h loading rate and RTCM prediction of secular strain.

(ii) *Comparison with creep data*

The data for the Oldorp 1-1 experiment shown Figure 11 include an interval of creep at the constant effective stress $\sigma'_c = 323\,\mathrm{bar}$. The data recorded during 8 days of creep have been fitted by a logarithmic creep strain of the form $\varepsilon = \varepsilon_c + c_1 \ln(1 + c_2\, t)$. An extrapolation of these data beyond 200 hrs of observation is compared in Figure 13 with the creep response predicted by equation (45). The agreement is quite close, with the exception of an irrelevant first day. It depends however quite sensitively on the value of the exponent ‘b’, and since this is the only candidate for a temperature-sensitive parameter, the possibility of a very different outcome of this comparison at reservoir temperatures cannot be ruled out.

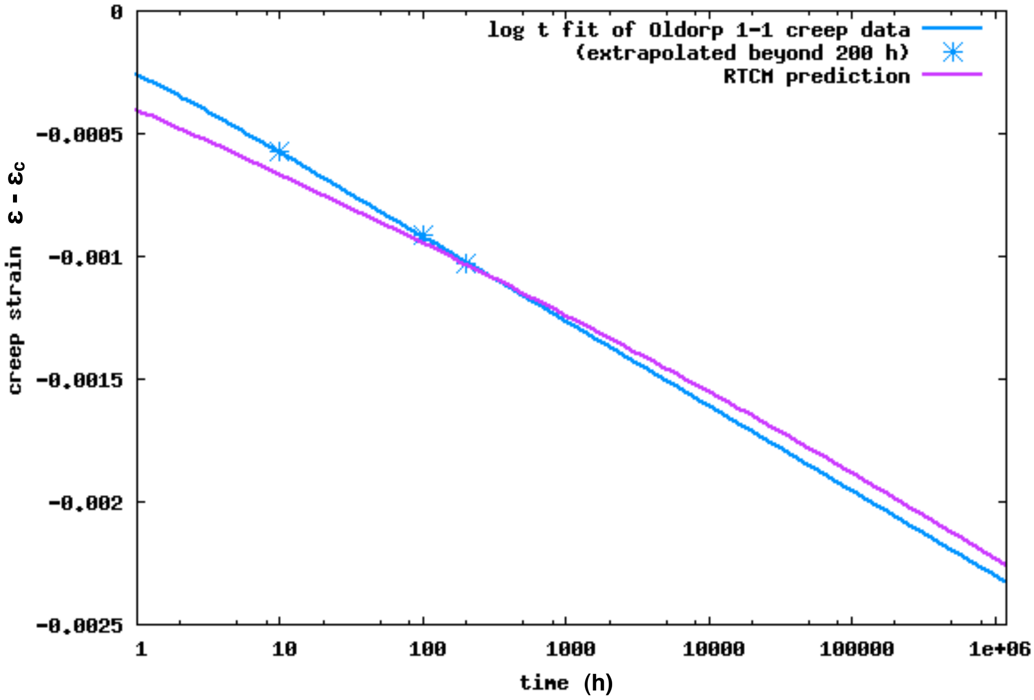


Figure 13. Comparison of uniaxial creep of Oldorp 1-1 core with theoretical prediction of RTCM. Experimental data at $\sigma'_c = 323\,\text{bar}$ axial effective stress were fitted by blue line $\varepsilon(t) = \varepsilon_c + c_1 \ln(1 + c_2 t)$, with $\varepsilon_c = 0.0132$, $c_1 = 1.5 \times 10^{-4}$, $c_2 = 4.59$, and extrapolated beyond 200 hrs. RTCM prediction of creep strain by eq. (45), with $\varepsilon_c = 0.0132$, $c_{m,prod} = 1.37 \times 10^{-5}\ \text{bar}^{-1}$, $\dot{\sigma}'_p = 62\,\text{bar/h}$, $b = 0.024$.

We conclude this evaluation of the RTCM model with the following observations:

- The RTCM isotach formulation employs equation 34 in the above as a convenient, universal scaling relationship that requires only two isotachs (linear data trends of strain vs stress) - recorded at different but constant loading rates either in the lab or field (preferentially) - in order to predict the isotach for any other loading rate.
- Given two such distinct isotachs and the initial *in situ* stress σ'_{ref} in the field, the depletion interval $\Delta\sigma'$ over which compaction & subsidence will be delayed can be estimated, using the RTCM isotach formulation, without the need to specify a fictitious (meaningless) "geological loading rate". (Sedimentary rock usually undergo diagenetic changes that alter their constitution to an extent that invalidates any

assumption of a uniform mechanical response throughout geologic time.)

- Following a discussion of the basic structure of the RTCM isotach formulation, we have compared predictions by the RTCM with observations of compaction strains obtained, first for a constant loading rate and secondly for creep at a constant stress. In these comparisons we took advantage of closed-form solutions, in the first at the expense of disregarding elastic strains. In the case of creep, our solution vindicates an earlier result by de Waal that has been criticized by Pruiksma *et al.* In both cases, and despite having obtained reasonable fits of an experimental data set for core material from the well Oldorf 1-1, certain fundamental doubts concerning the validity RTCM remain. These concern, both the theoretical structure of the model as well as its experimental verification, as summarized hereafter.
- The RTCM rests on an extremely narrow experimental base. In particular, there appear to be no experiments in which the loading rate was kept constant up to a maximum anticipated load; nor has the existence of linear isotachs been established in uniaxial deformation (not loading) experiments for a range of initial *in situ* stresses, i.e., different initial effective confining pressures, let alone the assumed convergence of such isotachs towards a single zero-stress intercept ε_0. Also the scaling relation (34) and the limits to its validity ought to have been explored experimentally in much more detail. As we argued in the above, (34) should probably not have been extended, as in [1], to include the elastic, rate-independent response, but remain restricted to a scaling relationship for the rate-dependent part of the total compressibility. The most important unanswered question remains however that of justifying the use of (34) in extrapolating from laboratory to field conditions, in particular, to much lower loading rates and higher temperatures. Experiments performed at reservoir temperatures for a sufficiently broad range of loading paths are indeed indispensable for a qualitative and quantitative understanding of compaction, in particular also of rate effects. It goes to the authors' credit that they express these reservations in the Conclusions of their report, although more than one reader must feel like being left in the cold when, in their final sentence, they caution that "Care must be taken in applying the derived models to the field scale."
- The RTCM is incapable of dealing with three-dimensional states of stress and strain. There remains therefore the important task to set up a general theory that will incorporate the experimental observations that have motivated the RTCM in its present form. Needless to say one should always demand of such a theory to re-

duce to classical poroelasticity in the limit of negligible rate effects, especially since an assumption of small strains is likely to suffice in the present context. This will require an awareness of relevant experimental work and ability to integrate substantial theory developments since the 1980's, especially of those focussing on pertinent micro-mechanisms of rock deformation. In other words, a task for theoretical mechanics/material science experts that have stayed close to innovative experimental work, both difficult to come across. A guiding thought would have to be not to cripple a theoretical description until it resembles one or the other 1D experiment, but to take clues from such an experiment for the development of a more general theory capable of dealing with 3D applications, but also suggesting further, and most likely more ambitious, experimental tests.

- In view of these difficulties and uncertainties one may ask the obvious question of how to deduce the input required by a field application of the RTCM 'directly' from observations made in the field, or, more precisely, how to use observations made in field A (in an advanced stage of depletion) to come up with a prediction for field B (in an early stage)? This could well eliminate the greatest uncertainties associated with an extrapolation of lab observations to the field, and would therefore appear as the safest approach when trying to come to grips with the question of compaction delay.

5. Remarks on Alternative Explanations for Subsidence Delay

There have been many discussions of possible causes of delayed subsidence other than rate-dependent compaction of the reservoir rock. Smits and de Waal [11] have analyzed a pressure-lag effects caused by the retarded depletion of shales embedded in more permeable reservoir rock. By comparing theoretical predictions[16] with observations in the Bachaquero and Wilmington fields, the authors reached the conclusion that "Nonlinear field-compaction and subsidence behaviour cannot be explained by pressure-lag effects, and that nonlinear field behavior is mainly caused by stress-rate effects." While one could probably agree with the first part of this conclusion, the second part, which was motivated by a better curve fit of field data by an RTCM prediction reported elsewhere [12], cannot be accepted, since it excludes categorically all other possible explanations, such as the overconsolidation behaviour that has often been considered as an explanation for the Bachaquero observations, or more generally, 3D stress changes leading to enhanced inelastic deformation after reaching a condition of failure.

Hettema *et al.* [13] have discussed possible and impossible causes of subsidence delay, among the latter the curious idea of an inertia effect and the misnomer of a "stress arching" effect[17]. They argue against pressure transients across a reservoir as a cause of subsidence delay on grounds that such transients require much less time than observed subsidence delay times. Similarly, they concur with the conclusion of Smits and de Waal [11] that depletion-lag effects will remain insignificant as long as low-permeability sections do not make up a significant part of the reservoir volume. In claiming some validity in general, both arguments imply however certain sedimentological and structural assumptions regarding the continuity of more permeable sections or the absence of significant reservoir

[16] Their prediction was based on 1D linear consolidation theory applied to a shale/sand sequence in which a given pressure drop $\Delta p_{sd}(t)$ in the sands induced a delayed pressure drop $\Delta p_{sh}(t)$ in the interspaced shale beds.

[17] By mentioning "stress arching" these authors (and others) suggest a genuine arching phenomenon, in which a loss of support of the overburden by the compacting reservoir is compensated by an increase in 'horizontal' stress within a overburden 'arch'. Such an interpretation of compaction-induced stress changes would be quite wrong, however, as long as these stress changes are of the kind predicted by poroelasticity theory, e.g., as in the work of Geertsma [14], Segall [15], Lehner [16], and many others; for the simple reason that these poroelastic *stress changes* come on top of an initial state of stress that is in equilibrium with gravitational body forces, and therefore satisfy an equilibrium condition from which body forces are absent. Here the well-known analogy between poroelasticity [17] and thermoelasticity [21] helps to visualize these stress changes in that they are exactly of the kind occurring in the surroundings of a region of cooling in a thermoelastic material, i.e., a *contracting* region just like a depleting reservoir! This will predict, for example, that reservoir compaction must induce a similar build-up in 'horizontal' stress above and below a reservoir, which would be impossible to explain by a genuine arching effect.

compartmentalization, be it by sealing faults or–on a much finer scale–in the form of lens-shaped sedimentary bodies that may be enveloped by low-permeability shales. The argument therefore seems to be one that cannot be fully relied upon in a particular case without support from actual field data.

Hettema *et al.* also mention the need to consider possible delay effects caused by one or the other aspect of depletion-induced overburden deformation. This question appears to have been investigated far too little. Suffice it here to mention different types of rate-dependent behaviour of the rocks in the reservoir surroundings, such as a genuine rate dependence as is known for rock salt, but also diffusion effects associated with the equilibration of undrained and drained pore pressures in different parts of the reservoir surroundings. The forgotten pioneering work by A. Scott Williamson [22],[23] of compaction/subsidence behaviour in stratified viscoelastic formations may be mentioned in this context, as they may possibly shed more light on the question of subsidence delay. Similarly, the broader, mixed analytical/numerical approach based on Maisel's formula [16],[20] might be of interest, if developed further to include heterogeneous, viscoelastic reservoir surroundings.

A further possible delay effect may reside in the nonlinear response of the topmost, soft sediments close to the surface, where vertical stresses are very low while horizontal stress changes may amount to a few percent of the decline in reservoir pressure [18]. Other, speculative, nonlinear effects may be due to the opening of 'horizontal' discontinuities of various kinds–for example weak bedding planes–in response to vertical unloading, a process that has been invoked to explain 4D seismic time-lapse observations [19]. Indeed, could the repeatedly observed reduced P-wave velocity above reservoirs be related to the phenomenon of subsidence delay?

The possibility of a delay effect inherent in the overburden response thus remains and cannot be excluded at this point without further study. It may have to do with pore pressure diffusion, with a linear or nonlinear viscoelastic response, or with other poorly constrained inelastic components of material behaviour. It seems clear, therefore, that future work will have to emphasize the study of material behaviour as much as 3D numerical model studies.

Concluding Discussion and Recommendations

The report under review begins with the following statement, which invites some immediate commentary below:

> This report presents a general formulation for reservoir compaction models developed for the calculations of the compaction of the Groningen field. Groningen, as well as other gasfields like Ameland, Anjum, Ezumazijl, Metslawier, shows a delay in the compaction at the starting phase of production. As production of the fields has been ongoing for several years, it is clear that the rate of subsidence has increased compared to the rate of subsidence at the start of production. This rate dependent compaction behavior has been observed in reservoir sands and sandstone in lab experiments, but has not been widely applied so far in field cases.

As first observation it must be said that the qualification 'general formulation', taken in any sense, is certainly inappropriate. In fact, the report almost totally ignores the experimental and theoretical rock mechanics literature of the past decades. It remains instead focused on one-dimensional constitutive models for three distinct types of rate-sensitive deformation behaviour; the first inspired by observations on soft soils and peat, the second the simplest textbook example of a linear viscoelastic solid (endowed with two springs and one dashpot), and the third–the RTCM–based on an empirical scaling relation whose range of validity remains unexplored and together with expected but unknown temperature effects makes it unsuitable for the prediction of field behaviour, except perhaps–and this is one "positive conclusion" of this evaluation–as a tool for correlating rate-dependent behaviour of distinct but lithologically similar reservoirs.

A further general remark, which must be made here, is that a delay in *reservoir compaction* (with respect to a declining mean reservoir pressure) is obviously difficult (and expensive) to confirm directly on a field-wide scale. Confounding it carelessly with an observed delay in *surface subsidence*, as happens in [1] in many places (cf. the above quotation), is therefore inadmissible in view of several other potential causes of delayed subsidence.

Pruiksma *et al.* also contend that "standard methods do not give satisfactory solutions for the measured subsidence" where "standard methods" presumably refers to the applications of poroelasticity theory in the work of Geertsma and van Opstal [14], Segall [15], and others, but excludes for example the work by Williamson [22],[23] on poroviscoelastic compaction of inhomogeneous (layered) reservoir rock and reservoir surroundings, needless to say in three dimensions. Another "standard" approach to the compaction-subsidence problem is of course offered by numerical FE solutions that place no constraints on reservoir geometry and material heterogeneity and will allow the implementation of a variety of nonlinear (rate-independent or rate-dependent) constitutive relations. Viesca *et al.* [24], for example, have studied the inelastic response of fluid-saturated rocks to near-by propagating earthquake ruptures, making use of constitutive descriptions for inelastic porous rocks than could serve as a model for analyses of depletion-induced stress changes in reservoirs, reservoir surroundings, and fault zones in particular. Several chapters in the book by Guéguen and Boutéca [25] also provide orientation on current (or recent) research into the micromechanics and continuum theories for fluid-saturated, elastic and inelastic porous rocks. Seen against this background we may sum up this evaluation of the models considered by Pruiksma *et al.* as follows:

1. The first two 'soft-soil' models originate from observations and the modeling of the deformation behaviour of clays and peat; they predict uniaxial creep even in the absence of a driving stress. The few successful fits of experimentally compaction data cannot eliminate this fundamental defect of theory. These models cannot provide reliable descriptions of creep and other rate-dependent deformation behaviour of sandstones and should therefore be rejected.
2. The SLS model should be replaced by a 3D linear poro-viscoelastic model of a general kind for which solutions can be obtained, either from the work of Williamson [22],[23] or in the form of an appropriate extension of Maysel's integral representation [16],[20]. (See also point 6. below.)
3. The characteristic scaling relation of the RTCM model links the rate-dependent compressibilities (or compliances) for different isotachs over an unknown range of loading rates. Failure to delimit this range and to determine temperature effects experimentally currently excludes the possibility of a direct extrapolation of laboratory observations to the field. This lack of experimental support and of a theoretical base suggests that use of the RTCM should be restricted to the correlation/extrapolation of field data among reservoirs with a substantial depletion history.
4. Isotach formulations may offer a certain computational advantage in dealing with uniaxial deformation and loading, which is the only mode considered by Pruiksma *et al.*. For 3D states of strain, which will have to be dealt with in general, these special loading and deformation paths are of no particular significance. Likewise,

experimental studies probing into compaction behaviour beyond a strictly elastic domain must usually explore different loading paths; and this will typically involve changes in stress (strain) rates, even if an axial strain (stress) rate is kept constant.

5. 3D model formulations are needed because depletion-induced stress changes and associated strains will usually involve all components of stress and strain. And while any fully three-dimensional theory will obviously allow for constant as well as variable loading-rate solutions, the reverse step from a 1D isotach model to a 3D application can in general not be made unless one is willing to disregard essential elements of a properly three-dimensional theory of material behaviour, elements that can only be identified and quantified by additional (e.g., triaxial or true triaxial) experiments under various loading conditions. Since this is well-known, the authors' approach appears to be based on the assumption that the modeling of compaction-induced subsidence can be adequately dealt with by treating reservoir compaction as a problem of uniaxial deformation. This cannot be correct in general in view of the multitude of possible reservoir geometries; even in the simplest case of a perfectly homogeneous, horizontal reservoir of uniform thickness there will be substantial depletion-induced changes in the principal stress ratios. We cite an experimental study, pertinent in this context: In conventional triaxial creep experiments on water-saturated Darley Dale sandstone Heap *et al.* [6] found axial strain rates to depend heavily on applied differential stress, i.e., a reduction of only 10% in differential stress resulted in a decrease in strain rate of more than two orders of magnitude. The results of their creep experiments show that the underlying stress corrosion process "*appears to be significantly inhibited at higher effective confining pressures, with the creep strain rate reduced by multiple orders of magnitude.*" It will be clear that this sensitive response may vary greatly among samples of the same type of rock, and will vary even more so when different deformation mechanisms come into play with different rock types. Therefore the application of soft-soil or granular media models to consolidated sandstones that deform by fundamentally different micro-mechanisms cannot be justified.

6. 3D analytical and semi-analytical models of depletion-induced stresses and strains within and beyond a reservoir are available and should always be employed as a screening tool of what to expect. Some of these 3D models make allowance for linear viscoelastic behaviour in the form of some general relaxation or creep function; in that case there always exists the rate-independent limit of a poroelastic behaviour.

7. A major problem faced by all models is that of translating laboratory results into a response under field conditions at higher temperatures, different pore fluid chemistries, different stress paths, and smaller (by many orders of magnitude) loading rates. This problem is indeed mentioned repeatedly by Pruiksma *et al.*, especially so in the last paragraph of the report Summary and Conclusions, although there is little comfort to be had by a potential user of the proposed models from the authors' final warning that "Care must be taken in applying the derived models to field scale"!
8. Studies of alternative or additional causes of subsidence delay, such as discussed briefly in Section 5 of the present report, should be continued, as long as these causes cannot be excluded.
9. In Europe, pertinent academic research in experimental rock mechanics is being carried out in several institutions. A list of these laboratories and their directors or chief scientists will be communicated as part of the present evaluation.

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[1] Regarding the first objective, the authors have done an excellent job in elucidating the connections among the four classes of models. With reference to the original formulation of *Suklje* (1957), the mathematical assumptions that allow the simplification to the second linearized and third SLS models were spelled out explicitly. The authors also establish clearly the connection of these models to the RTCM, which is by no means obvious without the development presented herein. Sections 5.3 and 5.4 seem to present results of prime importance to the funding agency, but I cannot realistically assess their claims (that both the NAM time decay model and TNO's Ameland model are intrinsically SLS model), because they simply have not provided sufficient details in the report for this purpose. The authors cited two key references: [1] is a TNO report that I don't have access, and [12] is not directly on the NAM model and includes only a peripheral discussion of this topic.

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Since the request was for the reviewer to pay “particular attention for the applicability of the isotach formulation of the RTCM to the compaction behavior of well consolidated sandstone”, an important question that should be posed is whether there are critical gaps not being addressed in the kind of comparison being undertaken in the report, and if so, how can they possibly be remedied. The comparison here is meaningful only if the sandstone cores are representative of the reservoir setting and the applied stress state in the laboratory pertains to that in the field. Extensive rock mechanics studies have documented that elastic and inelastic behavior of sandstones are sensitively dependent on attributes including mineralogy, porosity and grain size (*Wong & Baud*, 2012). Presumably in the particular reservoirs of interest here, an extensive data base has been accumulated. It is therefore important to ask: how were the two cores selected for the comparison here, and to what extent are they representative of the reservoir rocks here?

A second gap is related to the fixation on uniaxial strain testing. Notwithstanding its convenience, an oedometer has an intrinsic limitation in that mechanical deformation is characterized with knowledge of only the axial stress and no information whatsoever on the transverse stresses. With an applied stress state that is ambiguous and difficult to interpret, it is very difficult to develop from such data a realistic constitutive model, especially for a consolidated rock. Therefore a necessary remedy is to undertake a more systematic investigation in the laboratory of the influence of deviatoric stresses on a larger number of representative sandstone samples.

The authors seem unaware of extensive studies of the brittle creep behavior that have appeared recently in the literature. A good place to start would be the review of *Brantut et al*. (2013). Although such data are typically presented in term of the creep responses and not necessarily in the isotach framework, the authors have actually laid the foundation here in that they have established in section 8.1 the connection between the RTCM parameters and standard creep parameters. It is likely that very useful insights can be gained from scrutiny and synthesis of some of the published data which are of very high quality. For example, one conclusion may be that the RTCM parameters cannot be idealized as constant, but are sensitive to the stress state.

[3] Given some of the above concerns regarding comparison of the model prediction with very limited and possibly unrepresentative laboratory data, it is of utmost importance to first address them before venturing to extrapolate the models to field setting. The authors have carefully listed a number of additional limitations (including temperature, fluid, and difference in strain rate space) which should also be kept in mind.

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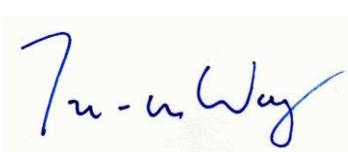
Reviewer:



Teng-fong Wong (March 31, 2014, Hong Kong)

N O T E

Restricted

DATE 21 May 2014

FROM Antony Mossop NAM-PTU/E/Q

TO Jan van Elk NAM-UIO/T/DL
Dirk Doornhof NAM-PTU/E/Q

SUBJECT NOTE ON TNO REPORT 'A GENERAL FRAMEWORK FOR RATE DEPENDENT COMPACTION MODELS FOR RESERVOIR ROCK' (**TNO 2013 R11405**)

Introduction

This note summarises the reviews by two independent external reviewers and one internal NAM reviewer of the TNO report named in the title.

Review Summary

One of the main stated objectives of the TNO report, is to provide a general formulation for a number of one-dimensional, isotach type, soil/rock compaction relationships, allowing them to be more easily compared and contrasted. The authors have done a good job in achieving this objective and describing the framework clearly. There is little to criticize with respect to this aspect of the report, (although mention was made that the comments in the last part of section 8.1, with regard to being unable to treat compaction coefficient as a constant, are seen as unwarranted, as temporal integration will yield an effective constant value).

The 1-D isotach type compaction relationships were then compared to compaction measurements made on two laboratory samples. This part of the report is seen as somewhat less satisfactory. There is little information given, both as to the experimental data itself, and the steps taken to fit, or invert for, the various free parameters. It is also noted that it is hard to assess the claim that the NAM time decay model, the TNO Ameland model and the SLS model are all intrinsically the same, as there is insufficient detail provided to do so. Of greater concern is the fact that the laboratory data being used to test the validity of the various compaction relationships, were also originally used in formulating one of these relationships (RTCM); unsurprisingly this same relationship is then shown to be the compaction relationship that provides the closest fit to these data. This is a tautology and undermines any claims as to these comparisons being objective tests.

The physical implications and 'reasonableness' of the various compaction relationships raise a number of concerns. The introduction of 'state parameters' without a clear mechanical motivation or definition as to their precise nature in the isotach and RTCM relationships is questionable. Similarly, the isotach and RTCM relationships seem to have no possibility of a state of static equilibrium, and even appear to predict that deformation occurs indefinitely even when there is no applied stress, while this is perhaps a natural outcome of the asymptotic functions involved, it is physically troubling. In the RTCM relationship, the role of the initial/geological/reference compaction stress rate is difficult to interpret, it appears to behave as a material property that governs the compaction response of the material forever after, which implies a permanent imprinting or memory of this particular stress rate that dictates all subsequent mechanical behavior, this is seen as implausible. It is also noted that, typically, more refined physical theories, should reduce to, or at least approximate to, known, proven, simpler theories; the observation is that it is hard to see how the isotach type laws could be simplified to a poroelastic law.

A major criticism by all the reviewers is that a 1-D uniaxial stress-strain relationship is simply insufficient to describe the constitutive behavior of a material, that the evolution of the transverse stresses has a profound impact on the mechanical response and cannot be neglected and left undetermined. The compaction relationships examined are essentially phenomenological descriptions of a particular type of deformation process and do not provide a sufficient constraint on the constitutive properties. That is not to say that such relationships are invalid, merely that they cannot be generalized to other deformation geometries without additional information or assumptions.

In a similar vein to the last stated point, the report authors themselves recognize that generalizing the laboratory results, and derived 1-D stress-strain relationships, to the reservoir scale of size, temperature, in-situ stress and production induced stress-rate is highly uncertain. It is in fact a standard tenet of viscoplasticity, that different deformation processes predominate, depending on pressure, strain rate and temperature, with widely varying constitutive relationships, relaxations times, etc. One of the reviewers notes that for the RTCM law, the only parameter that seems to be able to have potential temperature dependence is the 'b' exponent parameter. The report therefore, fails to make a convincing case that the best fitting stress-strain relationships, derived in the report, are directly applicable to the case of in-situ reservoir compaction, under very different conditions.

Recommendations

In terms of recommendations: the report highlights empirical observation based, 1-D soil mechanics compaction relationships and two laboratory results that exhibit time dependent deformation responses that significantly differ from the simple, first order, time dependent model, that is presently used for subsidence modeling – this discrepancy needs to be further investigated. While the report does not make a convincing case for abandoning presently applied time dependent models of reservoir induced subsidence, it does raise serious questions that cannot be easily dismissed. In their present form, the preferred isotach type stress-strain relationships, derived in the report, are not seen as satisfactory constitutive models, but that is not to say that the 'aberrant' time dependent behaviours that they seek to represent can be ignored. At present, there is sparse laboratory data, it would therefore be necessary to carry out further laboratory tests, over a range of conditions (preferably close to those in-situ), and measuring all relevant stress data, to test whether such isotach type behavior really is a general observation at the laboratory sample scale. At the field scale, it is unlikely that the collected data is of sufficient resolution to unambiguously constrain or test for logarithmic isotach type behavior (section 8.4 of the report makes this readily apparent). However, it is possible that the greater temporal data sampling that is becoming more readily available, in conjunction with more monitored reservoirs reaching the end of their production lifetimes, that opportunities for field scale testing might present themselves, although it is not unlikely that uncertainty in other relevant data, model, and parameter values would mask any potential signal.

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Teng-fong Wong, 2014. Review of TNO report on “A general framework for rate dependent compaction models for reservoir rock” (TNO2013R11405). (See Appendix 2)

Appendix 1

Evaluation TNO 2013
R11405Lehner.pdf

Appendix 2

TfWong_Review.pdf

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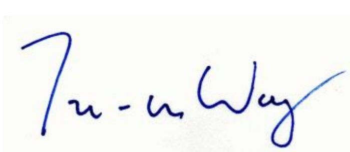

Teng-fong Wong (March 31, 2014, Hong Kong)

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One of the main stated objectives of the TNO report, is to provide a general formulation for a number of one-dimensional, isotach type, soil/rock compaction relationships, allowing them to be more easily compared and contrasted. The authors have done a good job in achieving this objective and describing the framework clearly. There is little to criticize with respect to this aspect of the report, (although mention was made that the comments in the last part of section 8.1, with regard to being unable to treat compaction coefficient as a constant, are seen as unwarranted, as temporal integration will yield an effective constant value).

The 1-D isotach type compaction relationships were then compared to compaction measurements made on two laboratory samples. This part of the report is seen as somewhat less satisfactory. There is little information given, both as to the experimental data itself, and the steps taken to fit, or invert for, the various free parameters. It is also noted that it is hard to assess the claim that the NAM time decay model, the TNO Ameland model and the SLS model are all intrinsically the same, as there is insufficient detail provided to do so. Of greater concern is the fact that the laboratory data being used to test the validity of the various compaction relationships, were also originally used in formulating one of these relationships (RTCM); unsurprisingly this same relationship is then shown to be the compaction relationship that provides the closest fit to these data. This is a tautology and undermines any claims as to these comparisons being objective tests.

The physical implications and 'reasonableness' of the various compaction relationships raise a number of concerns. The introduction of 'state parameters' without a clear mechanical motivation or definition as to their precise nature in the isotach and RTCM relationships is questionable. Similarly, the isotach and RTCM relationships seem to have no possibility of a state of static equilibrium, and even appear to predict that deformation occurs indefinitely even when there is no applied stress, while this is perhaps a natural outcome of the asymptotic functions involved, it is physically troubling. In the RTCM relationship, the role of the initial/geological/reference compaction stress rate is difficult to interpret, it appears to behave as a material property that governs the compaction response of the material forever after, which implies a permanent imprinting or memory of this particular stress rate that dictates all subsequent mechanical behavior, this is seen as implausible. It is also noted that, typically, more refined physical theories, should reduce to, or at least approximate to, known, proven, simpler theories; the observation is that it is hard to see how the isotach type laws could be simplified to a poroelastic law.

A major criticism by all the reviewers is that a 1-D uniaxial stress-strain relationship is simply insufficient to describe the constitutive behavior of a material, that the evolution of the transverse stresses has a profound impact on the mechanical response and cannot be neglected and left undetermined. The compaction relationships examined are essentially phenomenological descriptions of a particular type of deformation process and do not provide a sufficient constraint on the constitutive properties. That is not to say that such relationships are invalid, merely that they cannot be generalized to other deformation geometries without additional information or assumptions.

In a similar vein to the last stated point, the report authors themselves recognize that generalizing the laboratory results, and derived 1-D stress-strain relationships, to the reservoir scale of size, temperature, in-situ stress and production induced stress-rate is highly uncertain. It is in fact a standard tenet of viscoplasticity, that different deformation processes predominate, depending on pressure, strain rate and temperature, with widely varying constitutive relationships, relaxations times, etc. One of the reviewers notes that for the RTCM law, the only parameter that seems to be able to have potential temperature dependence is the 'b' exponent parameter. The report therefore, fails to make a convincing case that the best fitting stress-strain relationships, derived in the report, are directly applicable to the case of in-situ reservoir compaction, under very different conditions.

Recommendations

In terms of recommendations: the report highlights empirical observation based, 1-D soil mechanics compaction relationships and two laboratory results that exhibit time dependent deformation responses that significantly differ from the simple, first order, time dependent model, that is presently used for subsidence modeling – this discrepancy needs to be further investigated. While the report does not make a convincing case for abandoning presently applied time dependent models of reservoir induced subsidence, it does raise serious questions that cannot be easily dismissed. In their present form, the preferred isotach type stress-strain relationships, derived in the report, are not seen as satisfactory constitutive models, but that is not to say that the 'aberrant' time dependent behaviours that they seek to represent can be ignored. At present, there is sparse laboratory data, it would therefore be necessary to carry out further laboratory tests, over a range of conditions (preferably close to those in-situ), and measuring all relevant stress data, to test whether such isotach type behavior really is a general observation at the laboratory sample scale. At the field scale, it is unlikely that the collected data is of sufficient resolution to unambiguously constrain or test for logarithmic isotach type behavior (section 8.4 of the report makes this readily apparent). However, it is possible that the greater temporal data sampling that is becoming more readily available, in conjunction with more monitored reservoirs reaching the end of their production lifetimes, that opportunities for field scale testing might present themselves, although it is not unlikely that uncertainty in other relevant data, model, and parameter values would mask any potential signal.

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Lehner, F.K., 2014. An Evaluation of Report TNO 2013 R11405. (See Appendix 1)

Teng-fong Wong, 2014. Review of TNO report on "A general framework for rate dependent compaction models for reservoir rock" (TNO2013R11405). (See Appendix 2)

Appendix 1

Evaluation TNO 2013
R11405Lehner.pdf

Appendix 2

TfWong_Review.pdf

Date: December 14 2023

From: Peter Schutjens

To: Jan van Elk

Cc: [redacted]

Subject: TA1 due-diligence inspection of NAM work to review TNO report on subsidence delay and RTCM model

Here are my key observations and interpretations on the TNO report 2013 R11405 of Pruiksma et al: "A general framework for rate dependent compaction models for reservoir rock" (hereafter referred to as the "TNO report") in relation to the review of Dr. Florian Lehner: "An Evaluation of Report TNO 2013 R1140 (April 2014)", hereafter referred to as the Lehner-review. I have also read the comments of Professor Teng Fong Wong and of our colleague Tony Mossop included at the back of the Lehner-review.

1) The TNO report is a concise useful overview of four mathematical formulations of apparent rate-dependent behavior of sandstones. The report prefers RTCM as it seems to best describe one (1) set of experimental observations.

2) The Lehner-review is superior to the TNO report in that it dissects the mathematics of the four models in a more comprehensive, clearer and much more critical way. The underlying assumptions and non-physical consequences of e.g. the RTCM become more transparent in the Lehner-review than in the TNO-report. Indeed, the TNO report radiates the impression that the RTCM model is, indeed, the best model of the four, - able to describe apparent time-delay-effects in reservoir-compaction-induced subsidence, as interpreted for Groningen.

3) Taking a step back, the Lehner-review, using the same starting point and same experimental data as used in the TNO-report, coins that the SLS model, albeit simple, provides a fairly good description of the limited experimental data used in the TNO report (the Oldorp data). And because of its simplicity (based on linear visco-elasticity, i.e.,"dashpot analogy"), Lehner suggests it is to be preferred above the RTCM, as Lehner writes on page 16:

   "The SLS model expresses rate-dependent rock behavior as a cause of subsidence delay in simple and transparent way. It represents a textbook example of a rheological model of a linear viscoelastic solid. As a comparison with experimental data for an Oldorp 1-1 core shows, models of this kind, which feature only a single relaxation time cannot capture the spectrum of responses encountered in practice with very different loading rates. This deficiency is therefore likely to vitiate any attempts to apply experimentally determined model parameters, such as a relaxation time, to field conditions."

4) Lehner also notes the problem of a "geological strain rate" assumed in RTCM. This aspect of RTCM lacks a physical and geological basis, and as Tony Mossop also notes: RTCM predicts that compaction will continue indefinitely. Lehner also highlights a characteristic feature of the RTCM, which is the absence of a genuine material parameter in control of rate dependent behavior. And just like the other isotach models, it fails to correctly describe unload-reload behavior (stress cycling).

5) There is insufficient experimental data used in the TNO-report to test the validity and applicability of the four models. The Oldorp data-set has as reference "De Waal private communication" (without a date, *sic*), which is insufficient for a technical report. The TNO-report should either have limited itself to the theoretical aspects of the four compaction models, - or presented a comprehensive reliable dataset obtained on twin samples tested at different loading rates, and described in technical reports that can be traced. As Lehner and Wong note, there is plenty of such data available, published in books, reports and papers.

6) The macro-scale reservoir compaction and overburden response are driven by meso-scale and microscale mechanisms driven by physics, chemical reactions, and thermodynamic principles such as driving force and kinetics. Therefore, attention should be paid on describing, understanding and modeling the grain-scale mechanisms including detailed petrographical and microstructural analysis before and after compaction in the laboratory. Since intergranular pressure solution is too slow in quartz-water system at loading rates in laboratory and field to control deformation, attention should focus on grain-to-grain contact slipping, possibly combined with time-dependent microcrack growth, e.g., at the grain-to-grain contact where the highest normal stress and shear stress occur, and probably also the highest depletion-induced normal stress ***changes*** and shear stress ***changes*** occur. To this end, reversal of the direction of grain-on-grain slip direction from "geological" to "depletion-induced" deserves attention as a possible mechanism responsible for delay effects between effective stress increase and strain.

7) How to test the models, given the huge range of loading rates from experiment to field to geological loading rates? As the TNO report states at the end (p. 45):

   The analysis in this report is based on rate dependent behavior observed in lab experiments on sandstone core samples. A good match has been obtained with the RTCM model over the entire three orders of magnitude variation in loading rates in the experiments. However, it is uncertain if the observed trend may be extrapolated to field conditions. The smallest loading rate used in the lab experiments is about 1000 times higher than field pressure depletion rates and about a billion times higher than geological loading rates. Also, in upscaling from lab scale to field scale, physical processes other than the inherent creep of sandstone on a micro scale may contribute to the observed subsidence delay. Care must be taken in applying the derived models to field scale.

   Lehner comments on the same problem on page 17:

   Put in a simple way, rate-effects that are seen at extremely high loading rates in the laboratory will disappear under field conditions, while effects that appear in the field may never be observed in an experiment. [because the experiment would take too long, years to 10s of years].

   As Professor Wong notes, dedicated laboratory tests are required over a large range of strain rates. Mossop points to the opportunity to learn from field behavior, as more and more fields are approaching the end of full depletion.

8) Any further critical analysis of models should be preceded by a critical review of laboratory experiments done to investigate rate-dependent stress-strain behavior. There is much work done over the past decades, certainly not all applicable and useful, but certainly worth reviewing, if only to advice on the best way-forward in core deformation research for understanding rate-type effects, focusing e.g., on the stress and strain path (uniaxial strain) and the impact on rock deformation of the reduction in total horizontal stress (that accompanies

reservoir depletion, i.e., the “horizontal stress path coefficient”. For example, at the start of depletion, the relatively high mean total stress may favor elastic deformation with small strains per unit depletion, while with ongoing depletion and increasing shear stress, more inelastic strain will occur per unit depletion, possibly giving rise to an a similar “delay effect” as observed in experiments and field data.

9) The TNO report neglects a serious limitation of laboratory experiments: Frictional effects always occur in the experiment when changing the loading rate, due to O-rings and their VITON back-up rings having their own mechanical response. Therefore, only experiments with an internal load cell, placed as close to the sample as possible, should be used to test rock deformation models.

10) Hettema et al. analyze seven fields where subsidence-delay (compared to interpreted compaction) was observed, - and coin several mechanisms that could be causing this. The Hettema paper is not quoted in the TNO review. This is a short-coming, as any field-data set should be carefully analyzed, as the apparent “delayed” or “rate-dependent” compaction may have other causes than the continuum response of the matrix, such as pore-fluid pressure diffusion, slip along layer boundaries, slip along faults in the overburden; which could all occur in parallel, leading to “inertia” effects observed in other natural and man-made processes as well.

In summary:

- NAM has done the right thing by seeking a critical review of the TNO report which, despite its good intentions:

  a. fails to convince that RTCM can explain apparent rate effects in laboratory and field on a mechanistic basis, although RTCM may still have its practical use e.g., in the *description* of experimental and field data. As outlined by all reviewers (TNO, Lehner, Wong and Mossop) care must be taken in extrapolating RTCM outside of its tested range.

  b. does not point to further research, be it experimental, field-data-driven, or theoretical.

- I agree with the conclusions in the Lehner review, back-up by the comments of Professor Teng Fong Wong and Tony Mossop. In contrast to the TNO-report, these reviews give useful suggestions for research to proceed our understanding of reservoir compaction and the subsidence caused by it. Most importantly, next to continued field-data measurement and analysis, further research should be done using carefully controlled laboratory experiments on outcrop samples and twin samples from reservoir core, accompanied by petrographical and microstructural analysis. Such work should be preceded by a critical review of published laboratory deformation experiments that can help to understand the deformation behavior in reservoirs compacting because of hydrocarbon production.

Review by Anthony Mossop of:

TNO 2013 R11405

"A general framework for rate dependent compaction models for reservoir rock"

J.P. Pruiksma, J. N. Breunese and K. van Thienen-Visser

This report describes a number of quantitative, deterministic models, of varying complexity, that are proposed as potential candidate models for predicting the time/rate dependent compaction of gas reservoir rock formations, in the northern Netherlands, that are experiencing production induced pressure depletion. It places these models within a generalized formulation framework that allows them to be compared and contrasted with each other and, crucially, provides a practical approach to numerically compute predictions based on the 'Rate Type Compaction Model' [RTCM] originally proposed by de Waal [1986]. Further to this, the report attempts to determine the predictive capabilities of these models by comparing their predictions against the laboratory data collected and used by de Waal to originally formulate the RTCM model [1986].

Considering: the time that has elapsed since the original publication of this report; the number of reviews that have already been written and submitted about this report; and the fact that these models have already been adopted into the computer models used by NAM for subsidence prediction - I will limit this review to what I regard as the most salient points. I will also make use of context (this report was written at a time when these matters were being debated in some depth) and the benefit of 'hindsight'.

I shall start by stating that the general framework described in this report was a timely and welcome step forward in the practical computation of the RTCM, which was proving elusive at this time (circa 2013). The original published formulation of the RTCM [de Waal, 1986] was unusual/atypical in how it was presented, difficult to understand and somewhat open in terms of constraints and interpretation. This report by TNO cleared up many of these issues and provided a conceptual structure, based on one-dimensional, phenomenological models of soil compaction, that allowed the RTCM to be more easily applied and compared for the purposes of subsidence prediction.

However, there are a number of troubling flaws in this report.

1) A common criticism of the type of logarithmic, one-dimensional 'isotach' models as are described in this report, is that they are categorically *not* constitutive laws, as they violate frame invariance [Noll, 1954; Truesdell and Noll, 1965]. Such arguments are often, quite understandably, dismissed as being merely petty points of detail in situations where a simple, approximate, phenomenological model provides sufficiently accurate predictions within its applicable range [Gudehus, 2011]. That is clearly not the case here though. These 'technically wrong but pragmatic' isotach models are being extrapolated many orders of magnitude beyond their applicable range. The inherent problems associated with doing so cannot lightly be ignored, especially in situations where multiple parameters are weakly constrained by data such as occurs for the RTCM.

2) This then leads on to the next criticism I would make. The report is ostensibly about addressing issues of predicting _in-situ_ reservoir rock compaction, and the ensuing ground subsidence, caused by gas extraction. The evidence that there is a time and/or rate dependent mechanism involved in this process is sparse and on the edge of statistical significance, and certainly does not merit the application of a relatively obscure, far from widely accepted, five parametered phenomenological model, based on a single group of rock mechanics experimental data from one thesis. There is no motivation given as to why this should be chosen. If the purpose is to model the compaction of rock samples in a similar laboratory experimental set-up - which in some sections of the report appears to be the intent - then it would be perhaps arguable, but the report seems to vacillate back and forth on this point. With respect to modeling _in-situ_ compaction and subsidence though, the model is over-parametrized and the standard problems of model over-fitting would apply here, and are hard to ignore. A simple time decay curve (which could be of various functional forms, the standard linear model is perhaps the simplest, but not the only one) requires a single parameter, is far more physically supported, and would fit the field data equally well, and would be superior when judged from a statistical significance viewpoint.

3) Lastly, and I would argue of particular concern, is how the report judges the veracity of the various proposed models by comparing them against the very same rock mechanics experimental data that was used to formulate the RTCM in the first place. Not only is this highly questionable with respect to its applicability to what will happen in the field, where stress/strain rates are many orders of magnitude slower. It's also logically indefensible as a clear tautology [Wittgenstein, 1921] - it's quite simply inadmissible to reuse the same data that was used to formulate a given 'theory' and recast it as supposedly objective evidence to support said 'theory'. That this was not spotted or addressed is, to my mind, the most worrying flaw in this report.

To conclude - the report does an admirable job of presenting how to compute predictions based on the RTCM, but does not satisfactorily tackle the subject of when and where this would be applicable or sensible.

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A review of the Reviews by Lehner, Mossop, Wong and Schutjens
By Chris Spiers

There is general agreement amongst the reviewers that the TNO report provides a good overview of the four rate dependent models presented, and that the RTCM model appears to fit the experimental data considered well. At the same time, the significance of this fit is questioned, on the basis of a) the limited amount of experimental data considered and b) the fact that the RTCM formulation was inspired by the experimental data which it inevitably then matches (the tautology noted by Mossop). There is also broad recognition amongst the reviewers that the formulation of the RTCM model has no physical or mechanistic basis and indeed contains elements that are physically unrealistic, such as a non-zero strain rate at zero stress and an absence of temperature dependence. The fact that only 1D (vertical) stresses, strains and stress/strain rates are considered, neglecting lateral stress components and their variation through effects such as poro-elasticity, is also identified as raising serious uncertainties, or at least posing limitations. Several of the reviewers further observe that key experimental studies, not considered in the TNO report (by Brantut, Heap and co-workers), have already demonstrated deformation mechanisms that are highly sensitive to both temperature and lateral stress, albeit in sandstones with sub-reservoir porosities.

The key point emerging from all of the above, and emphasized by all reviewers (and by TNO), is that while the RTCM model fits the limited laboratory data considered by TNO quite well, extrapolation to in-situ stress/strain rates and P-T-stress-fluid conditions, using the RTCM, involves numerous uncertainties - though it is perhaps justifiable when constrained further by field measurements that better represent in-situ conditions. I agree with all of the above reservations raised by the reviewers.

In their conclusions, the reviewers call for improvements in modelling compaction through use of a wider range of existing experimental data, new experiments at realistic in-situ P-T-fluid and stress conditions, studies of the active deformation mechanisms, formulation of (3D) constitutive equations that account for these deformation mechanisms, and on-going testing/tuning against field measurements of compaction and subsidence. These are sound recommendations that have formed the underlying goal of NAM- and DeepNL-funded research on reservoir sandstone compaction in recent years, e.g. at Utrecht University and at Shell. However, results show extreme complexity in compaction behavior, including evidence for effects of sample damage, stress path, stress- and temperature-dependent changes in deformation mechanism (from compression and slip on intergranular clay firms to time-dependent grain cracking), and high sensitivity to microstructure (e.g. Hol et al., 2018; Pijnenburg et al., 2019; Pijnenburg & Spiers, 2020). The long duration of the experiments needed to approach in-situ strain/loading rates, even within 2-3 orders, also limits data acquisition. Data that are emerging from tests at true Groningen P-T-fluid conditions (e.g. Jefferd et al., 2021; Hangx and Pijnenburg, 2023) show broad consistency with the trends of the RTCM model, so far supporting its application in modelling reservoir compaction and subsidence. Unfortunately, a physical basis to constrain extrapolation to reservoir loading/strain rates is still lacking. On the other hand, no alternative model, empirical or microphysical, can yet be put forward that is demonstrably better than the RTCM or characterized by lower uncertainties in terms of fit to laboratory data or in the context of extrapolation (see Jefferd et al., 2021; Hangx and Pijnenburg, 2023). For the moment, then, the RTCM approach is difficult to improve upon, i.e. until ongoing research provides further constraints.

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Review of the reports by Florian Lehner, Teng-Fong Wong, Anthony Mossop and Peter Schutjens: On Rate-type compaction models for reservoir rock.

Mateo Acosta, Jean-Philippe Avouac

September 10, 2024

Contents

0 Introduction

The TNO report 2013 R11405: "A General Framework for Rate Dependent Compaction Models for Reservoir Rock," has been evaluated by multiple experts. The current document discusses the insights provided by Florian Lehner, Teng-Fong Wong, Anthony Mossop, and Peter Schutjens and presents additional elements based on our own assessment and some recent research results.

Forecasting surface displacement due to subsurface reservoir operations is a topic of major importance in the oil and gas industry. This topic is of particular relevance in the context of the Groningen gas field where significant induced seismicity has occurred in association with surface subsidence due to the reservoir compaction driven by gas extraction (Van Elk et al., 2017). Geomechanical models can be used to forecast how reservoirs deform during and after the operations period. All porous rocks exhibit a recoverable poroelastic deformation in response to fluid pressure changes. There is evidence that reservoir deformation might also be inelastic (de Waal, 1986; De Waal and Smits, 1988; Hettema et al., 2002). While poroelasticity is a well established theory with solid experimental and theoretical foundations, the understanding of inelastic deformation of porous rocks is not as advanced.

The Rate Type Compaction Model (RTCM), first proposed by deWaal and collaborators (de Waal, 1986; De Waal and Smits, 1988; de Waal et al., 2015), and its isotach formulation (RTiCM) (Pruiksma et al., 2015; van Thienen-Visser et al., 2015), provides a formulation of inelastic compaction derived from rock mechanics experiments. The TNO report shows that this model can explain successfully field observations of surface subsidence above depleting reservoirs. However, the reports by Lehner, Wong, Mossop and Schutjens show that the model and the its up-scaling to reservoir scale can be questioned.

Hereafter, we discuss the experimental and theoretical foundations of the RTiCM model, and of possible alternative models of inelastic compaction in light of the comments raised by the reviewers of the TNO report.

To provide a structure to the discussion, we follow a logical sequence of questions such that a consistent negative answer to the earlier question stops the progression to subsequent question as highlighted in Figure 1. A model is found adequate if able to clear through the logic diagram.

We complement the reports and reviewers comments with results from research conducted at the Center for Geomechanics and Mitigation of Geohazards at Caltech. Some of our results presented here are still preliminary as they haven't yet gone through peer review. We however included them as we think they bring useful insight to the debate.

In the following, the TNO report 2013 R11405 is called the "TNO report", the review by Florian Lehner is called the "Lehner report", the one by Teng-Fong Wong is called the "Wong report", the first one by Anthony Mossop is called the "first Mossop report", the one by Peter Schutjens is called the "Schutjens report", and the second one by by Anthony Mossop is called the "second Mossop report".

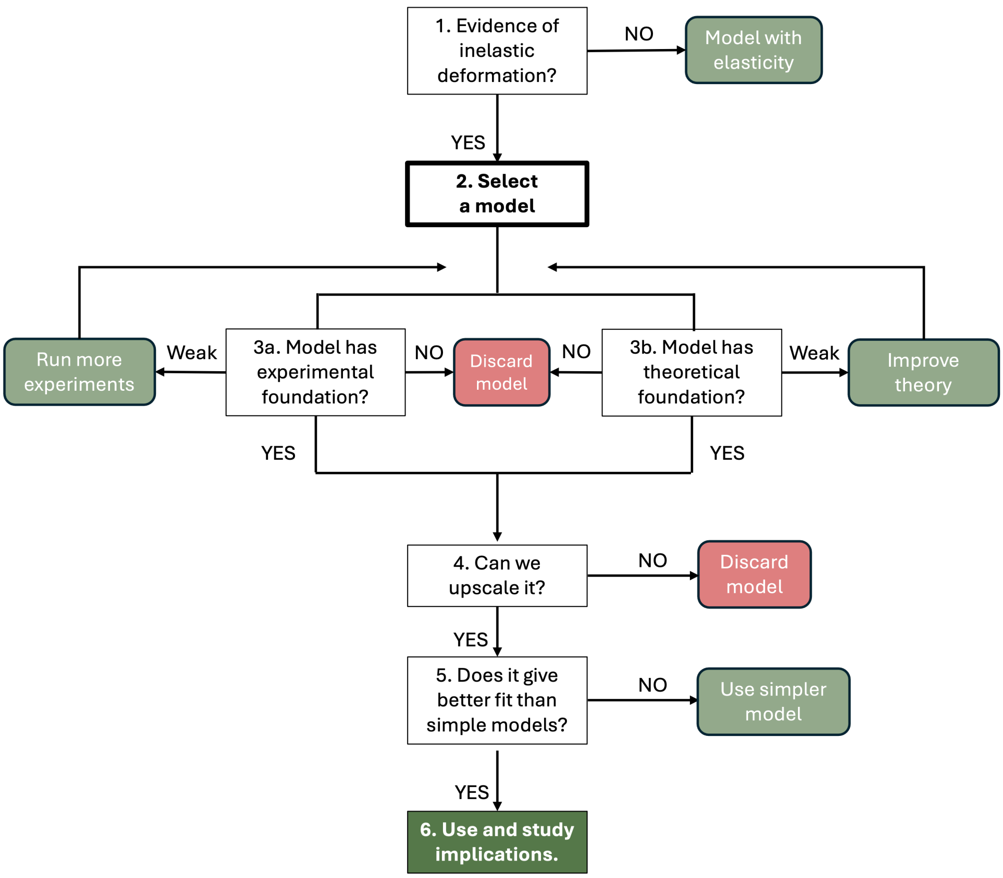

Figure 1: Logical flow chart used to structure the discussion presented in this report.

1 Do depleting reservoirs present conclusive evidence of inelastic compaction?

In the reports reviewed here, the authors seem to have taken for granted that the observation of a delay time between depletion and surface subsidence is a strong evidence for inelastic deformation. We therefore find it useful to revisit this premise. We consider that reservoir deformation is uniaxial (vertical) which is a commonly appropriate assumption (Geertsma, 1973).

1.1 Field observations

Inelastic compaction can be inferred from surface subsidence in two ways:

- **from a delay between observed subsidence and pressure depletion**. The observation of a delay between pressure drop in the reservoir, and the ground subsidence, suggest that the reservoir doesn't gradually adjust to the pressure change as expected from an elastic response (see Hettema et al. (2002)) for details). The delay can thus be the signature of time-dependent compaction. Note however that the reasoning assumes that surface subsidence at location (x,y) is equal, or at least proportional, to the local reservoir compaction at depth: the vertical strain integrated over the reservoir interval at location (x,y). This approximation is often incorrect as detailed below.
- **from time-dependent apparent elastic compressibility**. The apparent elastic compressibility is defined as the ratio of the vertical strain over the pressure drop. It quantifies how much the reservoir rock can compress under an applied stress. For a linear poro-elastic reservoir the compressibility should not vary in time or with applied stress (as long as the rock remains in the elastic domain). It is often determined from the slope of stress-vs-strain curves. If the reservoir experiences inelastic compaction, the apparent compressibility should change with time.

1.1.1 Observation of time delay between subsidence and pressure depletion.

Hettema et al. (2002) presents a review of case examples where surface subsidence increases non-linearly with the pressure depletion. It is relevant to start from this point as similar observations apparently motivated the development of the RTCM (de Waal, 1986). Hettema et al. (2002) analyzed measurements of subsidence over eight hydrocarbon fields. They generated plots of surface subsidence vs pressure depletion which show a systematic downward concavity. The authors fit a linear trend to the later portions of the data and extrapolate the trend to zero subsidence. The authors then infer a delay time marking the onset of subsidence along this extrapolated trend. The inferred time delay times range from 1.6 to 13 years for the eight fields studied. The measurements actually used in these plots are not described in details (whether subsidence and depletion were used as point-measurements or area-averaged). They conclude that inelastic compaction is required in these reservoirs. The evidence has several flaws: (i) Hettema et al. (2002) and de Waal (1986) seem to have selected only a limited dataset chosen for showing a downward concavity. We suspect that a broader dataset would show examples with no delay or where the subsidence vs pressure concavity is upward, implying a non-physical negative delay time.

In fact, while the relation between pressure depletion and surface subsidence might provide valuable insight, it cannot conclusively prove that depleting reservoirs consistently exhibit inelastic compaction. To illustrate this point we show in Figure 2a the subsidence vs pressure depletion plots for the Groningen gas field. The data were generated using a model which assumes a purely poro-elastic compaction of the reservoir, with spatially variable compressibility Smith et al. (2019). The reader is referred to Appendix A for more details regarding the modeling workflow used to generate these and the other modeling results included in the report. Due to the elastic deformation of the overburden, subsidence at any point of the surface is affected by the compaction of the whole reservoir, and not only at the point of the reservoir with the same location in map view. Due to pressure gradients in the reservoir and the heterogeneity in the spatial compressibility, it follows that the subsidence vs pressure plots are quite variable and non-linear despite compaction being linear-elastic (Figure 2a). At some points we observe non-linear curves similar to those of Hettema et al. (2002) with downward concavity. However, depending on the position where we sample the data, the curves can have a concave, near-linear, or convex shape. Therefore, if we were to use a linear fit on the later portions of the data, and extrapolate to zero subsidence to get a delay time, we would get negative, positive and null delay times depending only on where the data is sampled even though we have imposed a linear-elastic rheology for compaction. Such variability contrasts with the observations reported by de Waal (1986); Hettema et al. (2002). In any case, our results question that observed delayed subsidence derived from subsidence vs pressure plots can be considered as evidence of inelastic deformation. A non-linear relationship between the pressure depletion and the compaction derived from the inversion of the surface subsidence would provide a less ambiguous evidence. Such an analysis of the subsidence measured at Gronigen until 2016 is presented in Smith et al. (2019) but was found inconclusive.

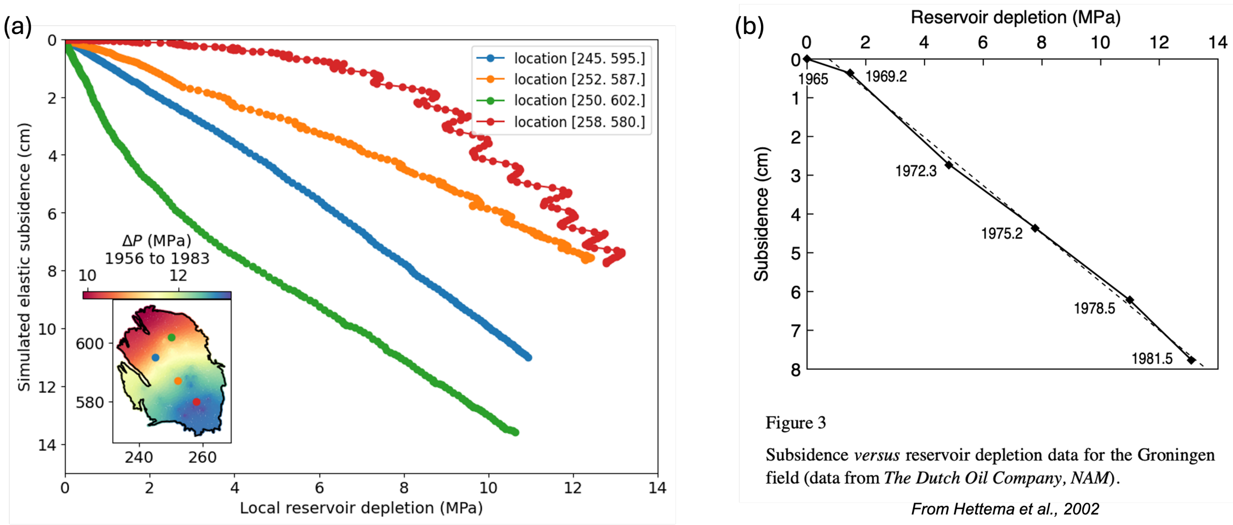

Figure 2: Surface subsidence versus depletion in the Groningen gas field. (a) Predictions for 1956 to 1983 from the GMG workflow (Smith et al., 2019, 2022; Meyer et al., 2022; Acosta et al., 2023) for poroelastic compaction in the reservoir using compressibility calibrated using data from 1960 to 2016 (see (Li et al., 2024) and (Smith et al., 2019) for details). We observe that even in the case of linear elastic deformation, the subsidence vs depletion curves are non-linear. The non linearity depends on the position of the sampling point and the heterogeneity of depletion and reservoir properties. (b) Figure 3 of Hettema et al. (2002). Observations and linear fit to the period (1969-1981) for the Groningen field. The position of the sampling points was not reported. This panel illustrates the method used to infer a time delay between compaction and depletion which served as justification for the development of inelastic compaction models. Comparison of panels (a) and (b) illustrates that careful and detailed analysis is needed to interpret subsidence-depletion curves.

1.1.2 Observation of time-dependent compressibility.

The recent work of Li et al. (2024), analyzes the time-dependent surface displacement above two reservoirs in northeastern Netherlands: the Norg Underground Gas Storage field (Norg in short) and the Groningen gas field (Groningen in short). In that study, the mechanics of reservoir compaction are investigated using extensive geodetic datasets, including InSAR (RADARSAT2, TerraSAR-X, Sentinel-1), GNSS, and optical leveling.

The surface displacement meaasure at Norg, shows distinctive seasonal fluctuations which closely follow the gas storage operations (NAM, 2016a). This signal has an amplitude of approximately 20 mm over a 5x5 km area (Figure 3). Using a purely poroelastic model (Smith et al., 2019; Li et al., 2024), we were able to match well the observed surface displacements with a single value of compressibility (6.2e-11 Pa^{-1}). A purely elastic rheology of the reservoir is thus a satisfying approximation in this case.

In the Groningen gas field, the surface displacement shows a more sustained subsidence which accumulated over several decades. The subsidence measurements additionally indicate spatially variable compressibility. These variations are resolvable due to the much larger extent of the Groningen gas field compared to Norg. By considering separate time windows and inverting for spatial compressiblity, we find that the elastic compressibility appears to have increased over time (Figure 4).

This observation requires either a non-linear elastic or inelastic compaction rheology of the reservoir. A non-linear elastic response could result from the compaction driven reduction of porosity. We would then expect a stiffening of the reservoir. The increase of apparent compressibility is therefore more likely due to inelastic compaction. The real compressibility would be obtained by using only the elastic compaction. If an inelastic component of compaction is present, the compressibility derived from the ratio of the total compaction to the pressure drop is an 'apparent compressibility' which actually overestimates the real compressibility which is meant to account only for elastic deformation.

We have shown that for two reservoirs located at approximately the same depths, but with different geometries and total depletion ($\Delta P \sim 12$ MPa for Norg (NAM, 2016a) and > 25 MPa for Groningen), the deformation behavior can appear either purely linear-elastic (Norg) or require inelastic deformation (Groningen). Because the two reservoirs were hosted in the same rock formation (Rotliegend), they must have the same rheology. A satisfying rheological model should be able to explain both the apparent linear elastic behavior at Norg and the non-linear behavior at Groningen.

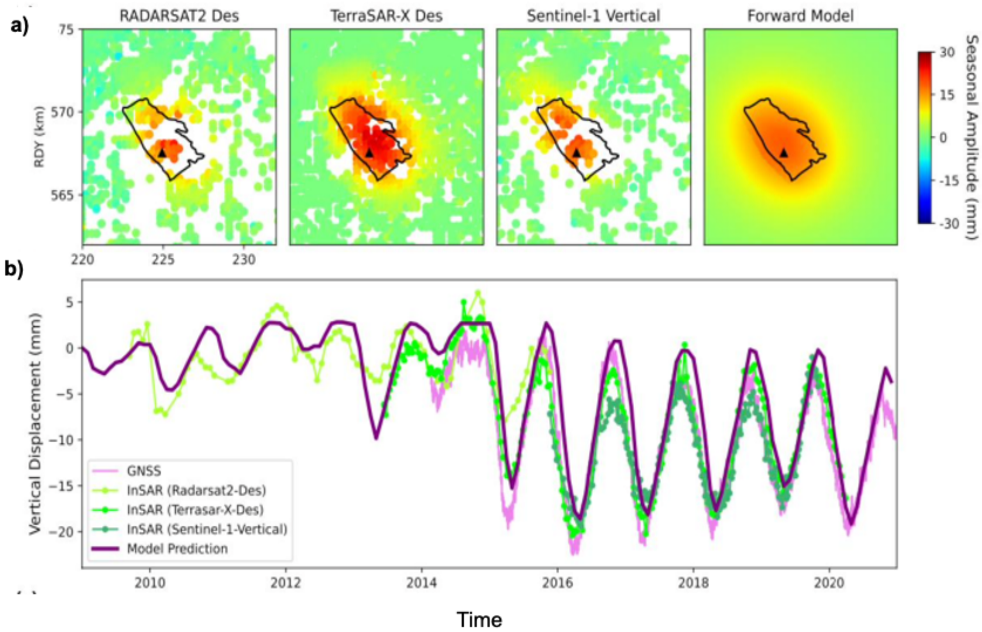

Figure 3: Surface displacement observed and modelled in the Norg-UGS field (from Li et al. (2024)). (a) Map of peak-to-peak amplitude of the seasonal oscillation of surface displacement over the Norg Underground Gas Storage field, retrieved from the Radarsat2, TerraSAR-X, and Sentinel-1 InSAR time series, along with the linear-elastic model model prediction. (b) Comparison of poroelastic model prediction with time series of surface displacement measured with GNSS, Radarsat2 InSAR, TerraSAR-X InSAR, Sentinel-1 InSAR. The location where the time series are extracted is co-located with GNSS station "norg" and shown as the black triangle in (a). For the forward model, the best-fitting c_m is 6.2e-11 Pa^{-1}. A purely elastic model with a fixed compressibility is sufficient to model surface displacement in the Norg UGS field pointing toward no inelastic dependent compaction.

1.2 Laboratory observations

Inelastic compaction can be observed in laboratory experiments under controlled conditions. Three regimes can be distinguished before macroscopic failure of a rock sample:

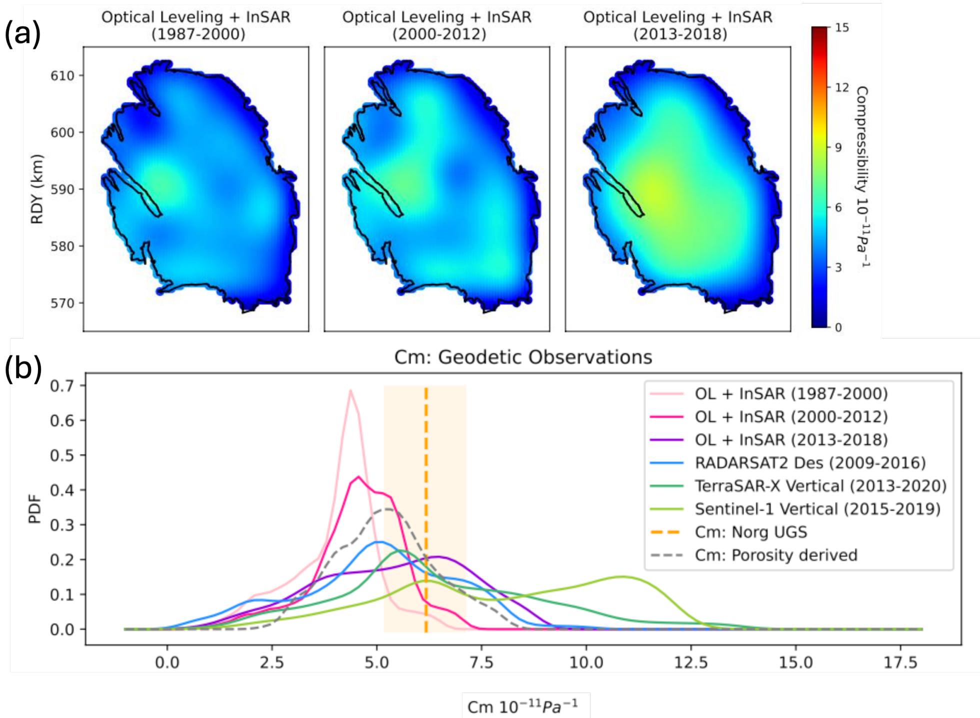

Figure 4: Apparent increase in elastic uniaxial compressibility (c'_m) in the Groningen field (Li et al., 2024) inverted from geodetic datasets spanning various time intervals. The increase of apparent uniaxial compressibility suggests inelastic deformation. (a) spatial distribution of c'_m for different time intervals (b) Probability density distribution of c'_m for different time intervals. The gray dashed line shows the probability density distribution of the compressibility derived from porosity (NAM, 2013). The best-fitting compressibility at Norg is shown for comparison (orange dashed line) with its uncertainty range (shaded organge area).

- elastic deformation occurs instantaneously upon the application of stress and is fully recoverable when the applied stress is removed. In the linear-elastic regime, the primary response recorded is the proportionality of strain and applied stress. These tests are often conducted at constant stressing-rate.
- inelastic (and possibly time -or- rate dependent) deformation at small strain (<1%). This regime is characterized by subtle, yet progressive accumulation of inelastic strain and generally does not lead directly to macroscopic failure (Hol et al., 2018; Pijnenburg et al., 2019a).
- inelastic (and possibly time -or- rate dependent) deformation at large strain (>1%) occurs over time and where rocks exhibit significant and irreversible deformation under sustained stress (creep, (Brantut et al., 2013)). The dominant microphysical process is subcritical crack growth at the mineral scale and is very sensitive to the environmental conditions (temperature and water content in particular) (Atkinson and Meredith, 1987; Brantut et al., 2013; Heap et al., 2009). In this regime, the rock will fail eventually due to unstable microcrack propagation if the applied differential stress exceeds some threshold.

The small-strain regime is the most relevant to depleting reservoirs. For example in the Groningen gas field, the pressure depletion ΔP can reach $\sim 25 \times 10^6$ Pa, and the maximum compressibilities inferred for the field can reach $c_m \sim 15 \times 10^{-11}$ Pa^{-1}, which would result in maximum strain of $\epsilon_z \sim 0.04$ % in the reservoir. This regime is also the least studied in the rock mechanics literature (Jaeger, 1979; Paterson and Wong, 2005; Jaeger et al., 2007). The RTiCM model was established based on experiments conducted in the small strain regime (de Waal, 1986; Pruiksma et al., 2015) as recalled in the next subsection. Other, more recent experiments, have led to different view of the microphysical processes involved in the this regime (Hol et al., 2018; Pijnenburg et al., 2019a,b; Pijnenburg and Spiers, 2020) as explained in the following subsection. It seems important to us to recognize that the processes operating in the small strain regime might be different that the one operating in the better studied regime at higher strain. The sensitivity of deformation rates to temperature, which is well documented for sub-critical crack growth (Heap et al., 2009; Brantut et al., 2013), remains unconstrained in the small-strain regime.

1.2.1 Rate-dependent sandstone compaction

de Waal (1986) developed the Rate Type Compaction Model (RTCM) based on rate-dependent compaction behavior observed in laboratory experiments conducted on sandstone samples and unconsolidated sands. deWaal's experiments were conducted mostly at room temperature, under oedometric stress conditions, with no fluid pressure, and at axial stressing rates ranging from 0.62 to 2300 bar/h. The RTCM formalism was formulated assuming that compaction is accommodated by grain boundary frictional sliding and adopting the rate dependency of friction observed in laboratory friction experiments (Dieterich, 1978; De Waal and Smits, 1988). This formalism is a steady-state version (Dieterich, 1978) of the now widely adopted and more general rate and state formalism which has been shown to represent well friction for a wide range of materials (Marone, 1998). Pruiksma et al. (2015) later revised the initial RTCM by introducing an isotach formulation (isotachs refer to lines of constant deformation rate at fixed stressing rate in the stress-strain diagram) which facilitates the implementation in numerical simulations.

The laboratory experiments of de Waal (1986), later presented in (Pruiksma et al., 2015), and again in (de Waal et al., 2015; van Thienen-Visser et al., 2015), demonstrate a stressing rate dependence of the compressibility of sandstone samples at small strain (<1%). In most (but not all of) these experiments, upon a change in stressing rate, the strain evolves first in a transient manner and then reaches an asymptotic strain rate if a fixed stressing rate is held for long enough (isotach), (Pruiksma et al., 2015; van Thienen-Visser et al., 2015).

However, the data set used in support of RTiCM is very scarce: in total, from (Pruiksma et al., 2015; van Thienen-Visser et al., 2015), and the Oldorp experiment presented in the Lehner report, only a handful of experiments conducted under very similar conditions show a clear isotach RTiCM behavior (all conducted at room temperature and without fluids). One might therefore question whether the model is representative of entire reservoir formations which might encompass rocks with various properties and under various conditions.

It is noteworthy that for a small range of laboratory stressing rates, and for experimental times manageable in the laboratory, distinguishing between parallel and non-parallel isotach slopes is a difficult task. For example Figure 1 of van Thienen-Visser et al. (2015) does not clearly show that the experimental data follows non-parallel isotachs due to (i) the sparse data presented and (ii) the poor match between the data and the model. This is a common feature of laboratory data supporting RTiCM models.

Another reason to question the up-scaling of RTiCM, as highlighted by all reviewers, is that the stressing rates used in the laboratory are, at minimum, 3 orders of magnitude higher than those during reservoir depletion (Figure 5), and in most cases, they are 5 to 6 orders of magnitude higher. If we extend the comparison to the low tectonic stressing rates in northern Netherlands, the difference is of ∼9 orders of magnitude. The validity of the RTiCM model over 9 orders of magnitude of strain rate is not warranted in absence of a theory or experimental data covering a larger range of strain rates.

Finally, all reviewers point to the limitation that the RTiCM was established at room temperature with no fluid pressure, so at conditions very different from in situ ones. It is known that, in the large-strain regime, inelastic deformation is drastically affected by the presence of fluids, their chemistry and temperature (Brantut et al., 2013; Heap et al., 2009). In the small-strain regime, Hol et al. (2018) found that for temperatures between 60°C and 100°C, at strain rates of 10 bar/h, temperature has very little effect on the amount of inelastic strain developed. The effect of temperature in these two regimes might in fact be contrasted. With the available laboratory data, it is questionable that RTiCM (de Waal, 1986; Pruiksma et al., 2015; Hol et al., 2018) is valid to model reservoir compaction at the field scale and in situ conditions.

1.2.2 Inelastic clay-film deformation

A notable set of experimental studies appeared in the literature after the reports and reviews discussed here were drafted (Hol et al., 2018; Pijnenburg et al., 2019a,b; Pijnenburg and Spiers, 2020; Verberne et al., 2021; F. Van Stappen et al., 2022). They shed interesting new light and we therefore mention them here. albeit without going in the details. These experiments were also conducted on samples from the Rotliegend sandstone formation and in the low-strain regime, but at conditions close to in situ conditions in terms of temperature and fluid pressure and chemistry. In addition, the experiments were complemented with post mortem microstructural observations. These studies suggest that reservoir deformation is partitioned between elastic strain and inelastic compression and slip of weak clay films coating the quartz grains in sandstones at stresses representative of the Groningen gas reservoir, but at stressing rates that can be a few orders of magnitude higher than those found during depletion.

These experiments would be a good basis for the development of a model of inelastic reservoir compaction. However they haven't yet been used to derive a compaction law that could be implemented in a geomechanical model. As with the RTiCM model, the extrapolation to natural conditions at much lower strain rates than those applied in the laboratory will be an issue.

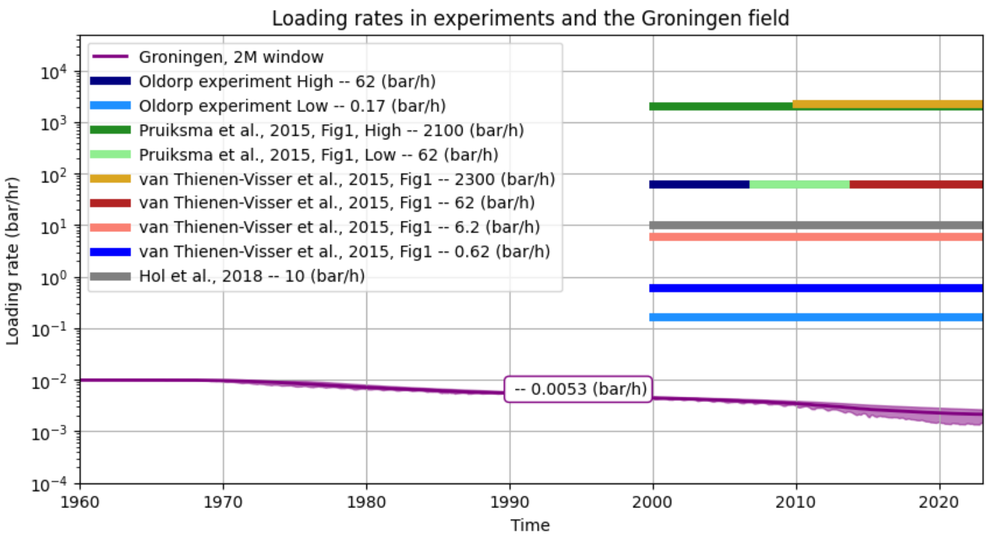

Figure 5: Vertical stressing rates measured in the Groningen gas field (from pressure depletion), and experiments (horizontal lines). We present the experiments by (de Waal, 1986) reported in Figure 1 of (Pruiksma et al., 2015) in green and light blue, and the experiments conducted in the Oldorp core from the reports reviewed here (dark blue and light green). The stressing rates computed for the Groningen gas field come from the pressure depletion model of Meyer et al. (2022) which is very similar to that of van Oeveren et al. (2017) in terms of pressure depletion. Stressing rates are computed using a 3 month moving window. The "geologic stressing rate" referred to by (Pruiksma et al., 2015) is not reported here but is expected to be many orders of magnitude lower than the minimum stressing rates during depletion in Groningen.

2 What models can we use to predict surface subsidence due to depletion of reservoirs?

It is customary and practical to assume uniaxial vertical compaction to model reservoir deformation and subsidence in real case applications. The approximation, which assumes a reservoir of large horizontal extent compared to their depth range, is actually often valid in real case examples (Geertsma, 1973). The RTiCM model which only deals with uniaxial strain can therefore be used in thin elongated reservoirs. As noted in the reviews of the TNO report this is however a limitation of the model. While 3D models are more general in their applicability, the computational cost in presence of non-linear rheologies can be prohibitively expensive. Moreover, the number of degrees of freedom in such models can impede a probabilistic analysis of the outcome and parameter space exploration. So the fact that RTiCM is restricted to uniaxial strain doesn't seem a severe limitation to us.

In the following, we avoid discussing the soft soil isotach and stress-linearized isotach models as these are merely building blocks in the TNO report for arriving to the RTiCM model, and the Lehner report deals with them in an extensive manner. We do highlight that, as stated in the Lehner report, these two models were derived for unconsolidated soils and as such their applicability to cemented sandstone reservoirs is unclear.

2.1 The Rate-Type Compaction Models

de Waal (1986) expresses total compaction (ϵ_z) as the sum of (i) elastic strain (ϵ_d, direct deformation) and (ii) a rate-dependent strain due to friction at mineral grain boundaries (ϵ_s, secular deformation):

$$\epsilon_z = \epsilon_d + \epsilon_s \tag{1}$$

In terms of applied effective stress (σ') and stressing rate ($\dot{\sigma}'$), the elastic ('direct') strain is defined as:

$$\epsilon_d = c_{m,a}(\sigma' - \sigma'_{ref}) \tag{2}$$

and the rate-dependent ('secular') strain is defined by its time derivative:

$$\dot{\epsilon}_s = \frac{\epsilon_z - \epsilon_{z0}}{\sigma'}.\dot{\sigma}'_{ref}.(\frac{\epsilon_z - \epsilon_{z0}}{\sigma'.c_{m,ref}})^{-1/b} \tag{3}$$

where ϵ_{z0} is a zero-strain intercept where all the isotachs converge, $c_{m,ref}$ is the compressibility at the fixed stressing rate $\dot{\sigma}'_{ref}$, c_{ma} is the direct compressibility, and b is a model parameter.

As detailed in the Lehner report the model produces non-parallel isotachs for different stressing rates (lines of constant slope in the strain-stress diagram). This observation is the basis for its RTiCM formulation (Pruiksma et al., 2015).

de Waal (1986)'s thesis compared the RTCM model predictions against field data from various reservoirs which potentially present inelastic compaction (see this report's section 1 for potential issues with the approach). The author claims that *"The obtained results, fully confirm[ing] the applicability of the RTCM under field conditions"*. As highlighted in the Lehner report (pp 25-26), the characteristic "delay-period" of the RTiCM model is not a material property but is inversely proportional to the stressing rate (equation 44 or Lehner report). The Lehner report criticizes this as a non-physical behavior but we note that this dependency allows to reconcile the very different characteristic day times observed in laboratory and and at the field-scale. The fact that RTiCM allows this upscaling is a notable merit of the formalism.

It is correct that RTiCM doesn't rely on a will documented microphysical mechanism. However it relates to rate- (and-state) dependent friction which has been shown to apply to most materials (Dieterich, 1978; Marone, 1998; Scholz, 1998). The RTCM model was indeed derived based on assuming rate-dependent friction at the grain contacts (De Waal and Smits, 1988). While rate-and-state dependent friction is an empirical formalism, hindering up-scalability to field situations, recent progress were made in the development of more theoretical foundations (Barbot, 2023; Putelat et al., 2011; Chen et al., 2017; Molinari and Perfettini, 2019; Aharonov and Scholz, 2018). So the rate-dependency adopted in RTiCM might have more general merit than acknowledged in the reviews of the TNO report.

Another strong criticism is that RTiCM predicts stress-free creep and this is non-physical. Again, this is a valid remark which discards RTiCM as a general constitutive model, but in the case of a deep buried reservoir stress-free conditions would not be a realistic condition of stress making the RTiCM a valid model under the geometrical and stress conditions of large gas fields. The fact remains that the model has some inconsistencies in terms of a purely constitutive model, but it seems reasonable for its use in gas reservoirs where the near uniaxial stress conditions and deep burial conditions would apply.

In summary, the main difficulties regarding RTCM, or equivalently RTiCM, are as follows:

- This model remains empirical, has little microphysical basis and the experimental data are insufficient to evaluate the sensitivity to environmental conditions.
- It is difficult to distinguish between parallel and nonparallel isotachs in experimental data. Doing so with field data seems like an impossible task given the narrow range of stressing rates.
- This model does not have the mathematical closed form of a constitutive model (highlighted in the Lehner report).
- Extrapolation to geologic rates, and in situ temperature from the laboratory data available is not warranted (highlighted by all reviewers).
- As the Mossop report points out, testing a model on the same data it was proposed is an 'unfair' validation. As such, the models should be tested against various datasets and other models tested against the datasets that helped derive the RTCM model.
- The RTCM model has not been tested against alternative, possibly simpler models.

2.2 The standard linear solid and time-decay models

Here, we discuss together the standard-linear solid (SLS) and time-decay (TD, (Mossop, 2012)) models due to their similarity. They both involve a single timescale of delay/relaxation which characterises the deformation response to a sudden stress change. This time scale is therefore a material property.

The SLS model is a linear rheological model. In its Kelvin-Voigt description, it consists of a spring (element a, of elastic compressibility $c_{m,a}$) in series with a parallel spring-dashpot element (with compressibility $c_{m,b}$, dashpot viscosity η, and a characteristic response time $T = c_{m,b}.\eta$). The total deformation (ϵ_z) is thus the sum of elastic (ϵ_a) and viscoelastic (reversible) compaction (ϵ_b). The Lehner report proposes that, if the timescale of the SLS model is inferred from field data, it could be the best candidate for forecasting subsidence.

The time-decay model (Mossop, 2012) describes inelastic deformation as a function of time, where the deformation rate decreases exponentially over time as stress is changed. This model is particularly relevant for predicting inelastic deformation in depleting gas reservoirs by capturing the delayed compaction that occurs depletion increases. In reservoirs with low-permeability formations (shales for example) in contact with high permeability ones (sandstones), the time-decay model can describe slow diffusion of fluids from low-into-high permeability formations; for example, the reduced permeability of the shale causes a time-lag in pressure equilibration, leading to prolonged and spatially heterogeneous deformation across the reservoir. Another interpretation would be the behavior of a linear viscoelastic solid where the system's response depends on the entire history of the applied stress. The TD model might thus be seen as more general model encompassing the SLS model.

The total deformation can be expressed, considering either the TD or the SLS model, as:

$$\epsilon_z = c_m.\Delta P * \frac{1}{T} e^{-\frac{t}{T}} \tag{4}$$

where c_m is the elastic rock compressibility, ΔP the pressure depletion, the symbol $*$ denotes convolution in time, and T is the characteristic timescale of the model.

The main limitation we can observe with these models is that, in their simpler formulation, they have only one characteristic timescale. As such, they are not able to reproduce both laboratory and field data. As highlighted in the Lehner report, the characteristic timescale for laboratory data needs to be lower than 2 hours while the measurement of subsidence at the field scale requires a characteristic timescales on the order of a few years (about 5 years inferred for Groningen subsidence by van Thienen-Visser et al. (2015), and about 3 years inferred by NAM (2013)). We have run a simple experiment to demonstrate that considering the characteristic time scale of relaxation to be a material property is an issue even when Norg and Groningen are compared, despite their similarity (same reservoir formation, same depth). The same rheological model should apply to the two reservoirs.

Reference	$c_{ma}/c_{m,ref}$	b	Lab-Field	Rock-Reservoir
Pruiksma et al., 2015	0.26	0.021	Lab	Sandstone
van Thienen-Visser et al., 2015	0.57	0.010	Lab	Sandstone
van Thienen-Visser et al., 2015	0.44	0.010	Field	Groningen
NAM, 2016	0.55	0.018	Field	Groningen

Table 1: RT(i)CM best model parameter inversion

Figure 6 shows a comparison between the strain computed using a linear elastic model (gray curve) and time-decay models (colored curves). As input, we used an oscillatory depletion/re-injection strategy of 8 MPa pore pressure amplitude around a mean of 32 MPa with a yearly period (NAM, 2016a). We use a compressibility $c_m = 6 \times 10^{-11} Pa$ (Li et al., 2024). This strategy is similar to the one observed in the Norg UGS field (Figure 3, NAM (2016a)) but oscillations are held for much longer to observe steady-state behavior of the strain response in all TD models.

The periods of the TD models include the characteristic timescales that were inverted for Groningen in the literature (∼5 years by van Thienen-Visser et al. (2015), and ∼3 years inferred by NAM (2013)). We notice that the strain response of TD models predicts a damped annual signal with a phase-shift of 2-3 months (Figure 6, bottom panel) compared to the prediction of the elastic model. To avoid any detectable phase shift and amplitude attenuation, as seen in the geodetic data of Norg ((Li et al., 2024), Figure 3) the time-decay model would need to have a material timescale of less than ∼1 month. This highlights an inconsistency between the calibration in Groningen and the observations for the Norg UGS.

A possible way to circumvent this issue would be to include several timescales in the TD/SLS models either by addition of multiple Kelvin-Voight elements with different characteristic timescales in the SLS or through multiple convolutions of time with various timescales in the TD model. If successful, such a model would suffer from the fact that the observation timescales during depletion are probably not long enough for an accurate forecast of long term relaxation. In simpler words, in the context of the SLS and TD models, we cannot calibrate timescales that we have not observed. This is a major difference with the RTiCM model. In any case, it is clear the TD (and SLS) models, in their simple form involving only one characteristic timescale, can be discarded.

3 Is the rate-type compaction model appropriate to model surface subsidence and seismicity?

In this section, we discuss the merits and limitations of the the Rate-Type Compaction Model in its isotach formulation (RTiCM) in view of the comparison with predictions from a simple poroelastic model of compaction (Smith et al., 2022; Acosta et al., 2023).

Both calculations are made using the same workflow (see Appendix) assuming either poroelastic compaction or a rate-dependent compaction using the RTiCM model. Table 1 list the parameters derived for laboratory experiments by Pruiksma et al. (2015), and van Thienen-Visser et al. (2015), or inverted for the Groningen field by NAM (2016b), and implemented by TNO (2022), and by van Thienen-Visser et al. (2015) (see Table 1). It shows that the parameters differ among studies and differ depending on the whether they are based on laboratory or field observations. In the RTiCM simulations shown below we use the parameters of NAM (2016b).

In section 1.1.2, we showed that for the Norg gas field, we did not need to account for any inelastic deformation to model surface subsidence due to cyclic injections/extractions and in the previous section that the apparent delay time due to inelastic deformation would need to be very short. This behavior points to inelastic deformation being negligible (not resolved with the measurements we have). Figure 7 shows that the RTiCM and purely linear poroelastic model predicts nearly identical surface displacement during the lifetime of the reservoir.

For the Groningen field, we show in Figure 8 the modeled surface subsidence averaged over the whole

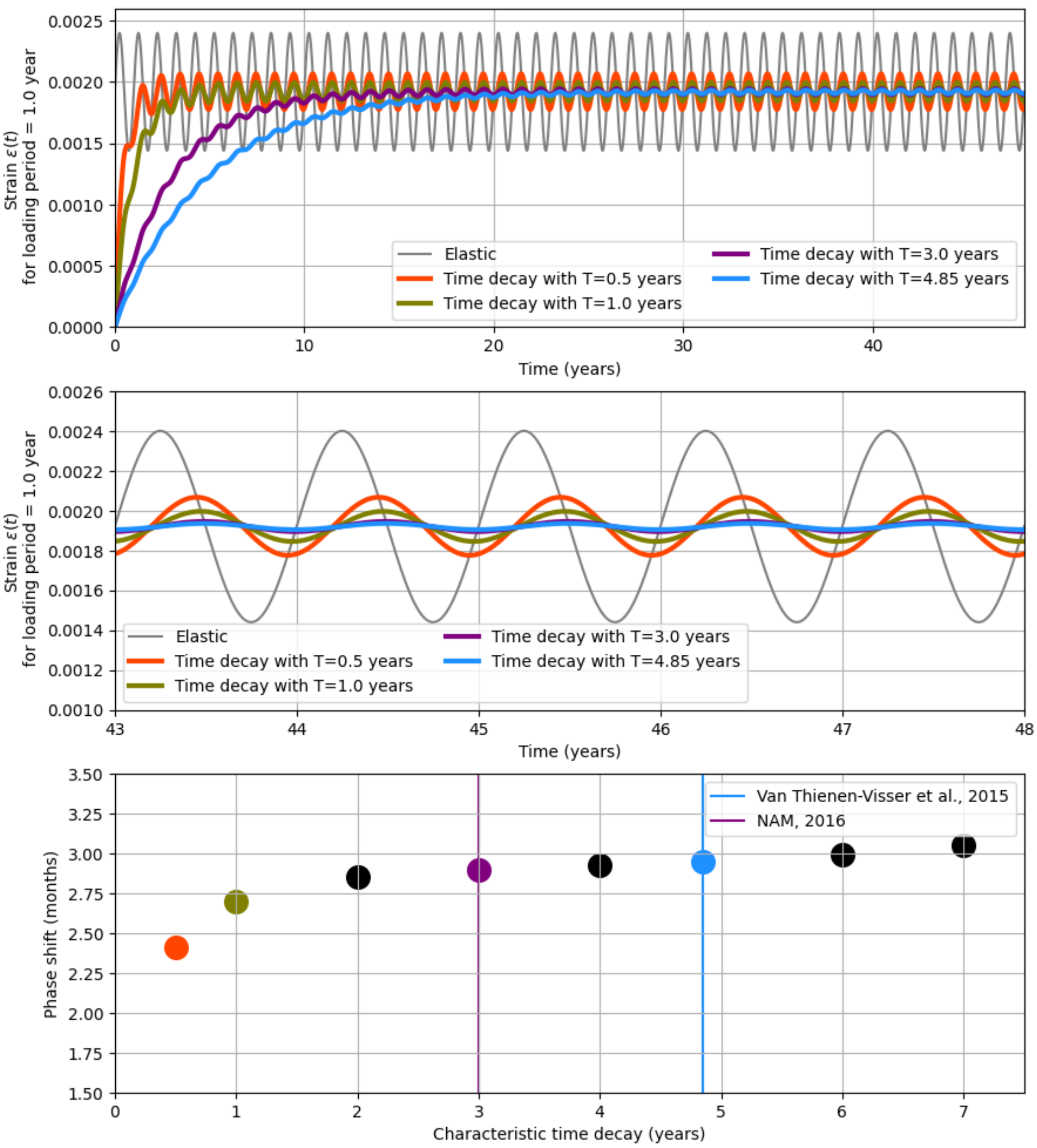

Figure 6: Analysis of the time decay model in the context of the Norg UGS field. We simulate an oscillatory depletion/re-injection strategy of 8 MPa pore pressure amplitude around a mean of 32 MPa with a yearly period (NAM, 2016a). To calculate strain, we use a compressibility $c_m = 6.2\times10^{-11}Pa$ (Li et al., 2024). This strategy is similar to the one observed in the Norg UGS field (Figure 3). Top panel: strain vs time for elastic deformation (gray curve), and for the time-decay model with different characteristic times (colored curves). Middle panel: zoom into the top panel when all the models have reached a steady-state oscillatory response. Lower panel: computed phase shift between the elastic and time-decay models function of the characteristic response time. Blue and purple curves correspond to the characteristic decay times for Groningen inferred by van Thienen-Visser et al. (2015) and NAM (2016b) respectively. All simulations presented here show a non-negligible phase shift (and amplitude ratio) between the elastic model and time-decay models, highlighting an inconsistency between the calibration in Groningen and the observations for the Norg UGS.

reservoir using (i) a linear-elastic reservoir rheology with an initial compressibility field calibrated from 1964-2016 using geodetic measurements (purple curve, (Smith et al., 2019); (ii) a linear-elastic reservoir rheology with a compressibility field calibrated from 1987-2000 using geodetic measurements (black curve, denoted early times compressibility (Li et al., 2024)); and (iii) an RTiCM reservoir rheology (as implemented by TNO (2022)) with a compressibility field calibrated as in model (ii) (light blue curve). Comparing both linear-elastic models, we notice that the model calibrated using a longer time span predicts more subsidence in the early days while it predicts less subsidence in the later period (after 2013). The RTiCM model predicts about 15% more subsidence by 2023 than its elastic counterpart, and about 8% more subsidence than the elastic model with longer calibration period. This comparison highlights that the calibration of the pure elastic model can absorb the effect inelastic deformation over the period of calibration but might then produce a biased forecast going into the future.

We show in Figure 9, the inverted apparent elastic compressibility from forward models generated using the RTiCM model calibrated on the early time span (1987-2000), similarly to what was done for Figure 4 using the geodetic data. We observe an increase in the apparent elastic compressibility of a similar magnitude than that observed in the geodetic data, meaning that this model provides a much better fit to the observations than simpler linear-elastic models given that the calibration does not adsorb inelastic effects. As such, we conclude that the RTiCM mode passes successfully stage 5 in the logic sequence of Figure 1 in that it explains well, qualitatively and quantitatively, the increase of apparent compressibility required by the subsidence measurements. We recall that the linear elastic model implies a constant compressibility and cannot explain the observation qualitatively.

These two observations highlight the merit of the RTiCM model at explaining inelastic deformation observed through surface subsidence in the field: In the Norg UGS field, we cannot distinguish between elastic or RTiCM deformation probably because the total pressure drop reached is about half than at Groningen and the reservoir stay depleted for a too short period of time for significant accumulation of inelastic deformation (NAM, 2016a).

Figure 10 now compares the seismicity forecast obtained assuming either the linear poroelastic compaction or RTiCM (stage 6 in the logic sequence of Figure 1). Our tests show that a combination of time-dependent compaction (RTiCM) and time dependent seismicity generation (Threshold Rate-and-State model of (Heimisson et al., 2022)) fits the data used for calibration as well as the linear poroelastic model, but no better (the root mean square error assuming RTiCM is 4.17 events, and 3.92 events assuming linear elastic compaction, Figure 10). The linear elastic model thus appears to perform slightly better at explaining the observed seismicity. Due to the inelastic component of the RTiCM compaction, the predicted seismicity rates beyond the period of calibration decays more slowly than predicted with the linear poro-elastic model. It would therefore be conservative to adopt the prediction of the RTiCM model for seismicity hazard assessment. We note that the impact of different rheologies on the total number of earthquakes predicted by 2035 is small. By 2035 using the average winter scenario of NAM (NAM, 2013), the RTiCM model predicts about 628 events of magnitude larger than 1.2, while the linear poro-elastic model predicts 618 earthquakes The total forecasted number of events (not accounting for field shut-in yet) is about 2% higher using the RTiCM-based seismicity model compared to using the linear-poroelastic based model which would translate in only a slightly higher hazard level.

4 Shortcomings and merits of RTiCM

Here we recall and comment the main criticisms of RTiCM provided by the Lehner, Wong, Mossop, and Schutjens reports. We generally agree with them that the theoretical and experimental foundations of RTiCM are limited and that RTiCM therefore lacks generality. We however find more merit in the RTiCM model than they do.

One of the main criticisms of the reviewers is that the RTiCM formulation accounts only for uniaxial deformation conditions whereas deformation constitutive models should account for triaxial conditions. While this criticism is valid in general for rock deformation models, our opinion is that if the matter at stake is to forecast deformation of thin reservoirs elongated in the horizontal planes, this limitation of the RTiCM model is of no consequence. It would be useful to carry out a systematic study to evaluate for what ratio of the horizontal length to thickness of the reservoirs (and which amount of strain) the uniaxial deformation approximation remains valid.

Another strong criticism is that RTiCM predicts stress-free creep, and this is non-physical. Again, this is a

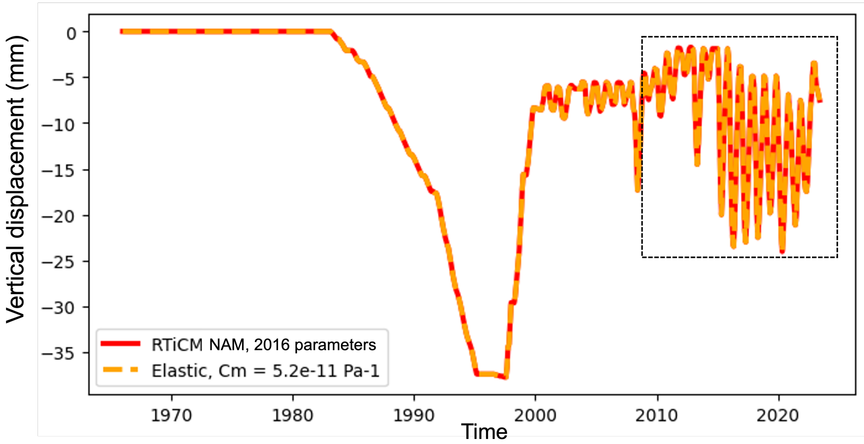

Figure 7: Averaged surface vertical displacement (negative for subsidence) within the Norg UGS reservoir from linear elastic and RTiCM model predictions using the calibration of (NAM, 2016b) for the RTiCM model parameters. Dashed square shows the data period shown in Figure 3.

valid remark and one reason the RTiCM cannot be considered a constitutive law. However this issue does not lead to a practical limitation. It is interesting that the standard formulation of Rate-and-state friction (Dieterich, 1978; Ruina, 1983) also leads to a non-physical inconsistency (friction becomes infinite for an interface with zero sliding velocity). The problem can be circumvented by adopting a regularized version of rate-and-state friction which was proposed based on thermodynamical considerations (Lapusta et al., 2000). It is well known that simulations of earthquake sequences are insensitive to this regularization and therefore the non-regularized version of the law remains widely used. A similar analysis of a potential regularization for the RTiCM model would be of interest.

The reviews (Lehner) in particular, raise the issue that temperature (and fluid chemistry) effects are not accounted for in the RTiCM, unless the power b would be temperature (and fluid chemistry) dependent. Based on our experience with friction laws, it is likely that the power b is actually temperature dependent. This topic would be worth an experimental investigation. In practice this limitation does not prevent from applying the model to real examples. If the model's parameters are estimated from geodetic observations (NAM, 2016b; van Thienen-Visser et al., 2015) they are then, by design, tuned to the in situ conditions.

Most reviewers are concerned that the TNO report didn't explore alternative models of inelastic compaction. This is certainly a valid concern. We note however, that the non-parallel isotachs observed experimentally are a feature that single-timescale SLS and TD models cannot reproduce. We are not aware of any standard compaction model that would have a good chance to pass these tests. In addition, we show that neither the Standard Linear Solid nor the Time Decay model would be able to reconcile the characteristic relaxation time observed in laboratory experiments, and in the subsidence from two fields studied in this report (Groningen and Norg).

The Wong report calls attention to compaction of depleting reservoirs being possibly driven by brittle creep. Brittle creep can indeed be a factor of inelastic deformation, but it is not clear that it should prevail in the small strain regime (see (Hol et al., 2018; Pijnenburg et al., 2019a,b; Pijnenburg and Spiers, 2020) for details). It is noteworthy that the mathematical structure of brittle creep type models (Brantut et al., 2013) is extremely similar to that of rate-and-state friction laws (Lockner, 1998; Perfettini and Avouac, 2004; Brantut et al., 2014). Since the RTiCM model is based on a rate-dependent friction (Dieterich, 1978; De Waal and Smits, 1988; de Waal, 1986), which is the steady-state version of the rate-and-state friction model (nowadays the a well accepted standard), it is probable that a compaction model based on brittle creep would lead to a formalism close to the RTiCM. This topic is worth some further investigation.

We fully agree with the problem highlighted by the Mossop, Lehner and Schutjens reports in that the same data used to derive the RTiCM model was used to show the closest fit to the data. This is a shortcoming of

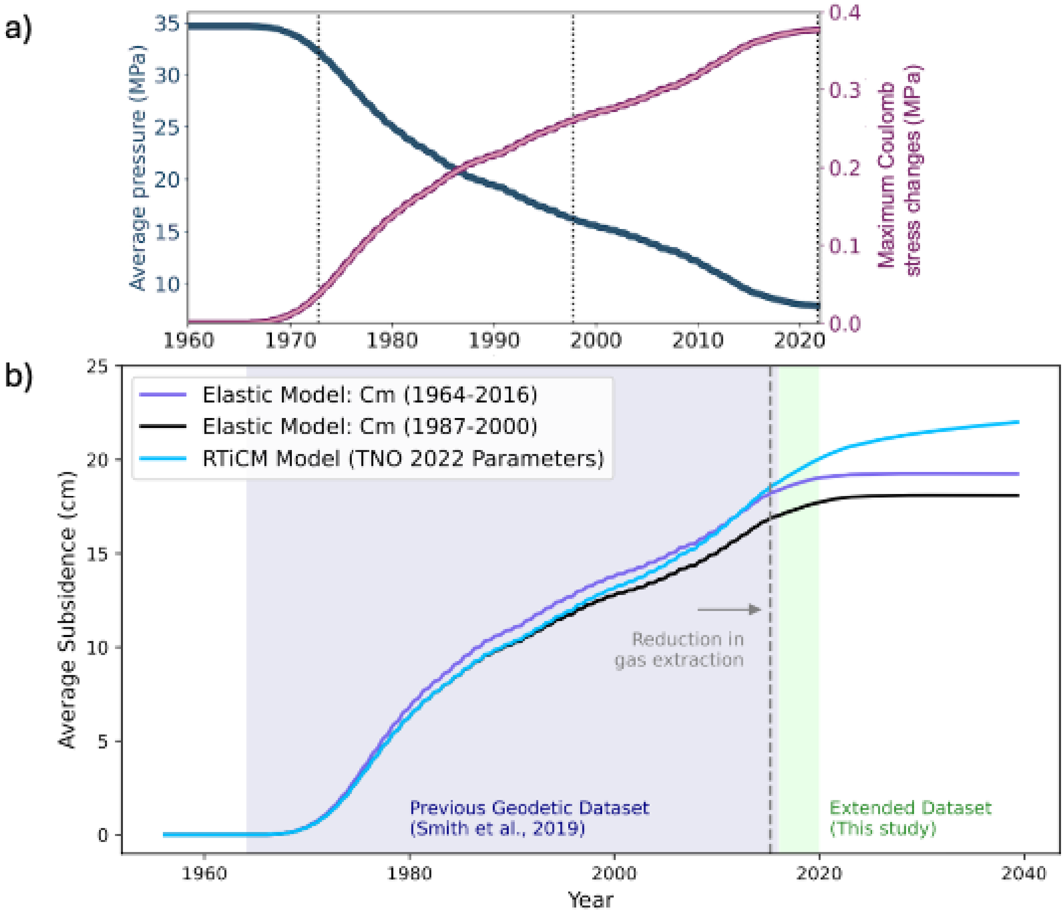

Figure 8: Modeling of the Groningen gas field compaction. (a) Average pressure (blue) and coulomb stress changes (pink) over time modeled for the Groningen field using the Flow2Quake workflow (modified from Acosta et al. (2023)) (b) Averaged surface subsidence within the Groningen reservoir from elastic and RTiCM model predictions (from (Li et al., 2024)). For the elastic models, we run the model using both compressibilities estimated from the entire period spanning 1964-2016 (Smith et al., 2019) as well as from the early period spanning 1987-2000, where there is little inelastic compaction. The shaded areas show the periods covered by the previous geodetic datasets as well as the extended datasets in (Li et al., 2024).

Pruiksma's work and the TNO report. In that sense, the RTiCM predictions should be used to explain other experimental data (now available in the work of Hol et al. (2018)). Probably new dedicated experiments are needed to this end. This is an important avenue to pursue.

Regarding the criticism on page 28 of the Lehner report, the issue of why creep laws (which closely resemble RTiCM for low values of the 'b' parameter) has been reconciled by (Brantut et al., 2014) in the large strain regime. There, the creep response depends on the deformation rate because the energy release rate affects sub critical microcrack propagation velocity under creep. The response also depends on the reached compressibility because it depends on the amount of strain in the rock which acts as a 'state' variable quantifying the amount of damage the rock has reached, facilitating crack propagation driven by a stress-or-energy deficit. It is noteworthy that the mathematical structure of brittle creep type models (Brantut et al., 2013) is extremely similar to that of rate-and-state friction laws (Brantut et al., 2014). As such, and because the RTiCM models were derived on the steady-state version of rate-and-state friction laws (Dieterich, 1978; De Waal and Smits, 1988; de Waal, 1986), it would be of interest to explore the if brittle creep models operate in the low-strain regime.

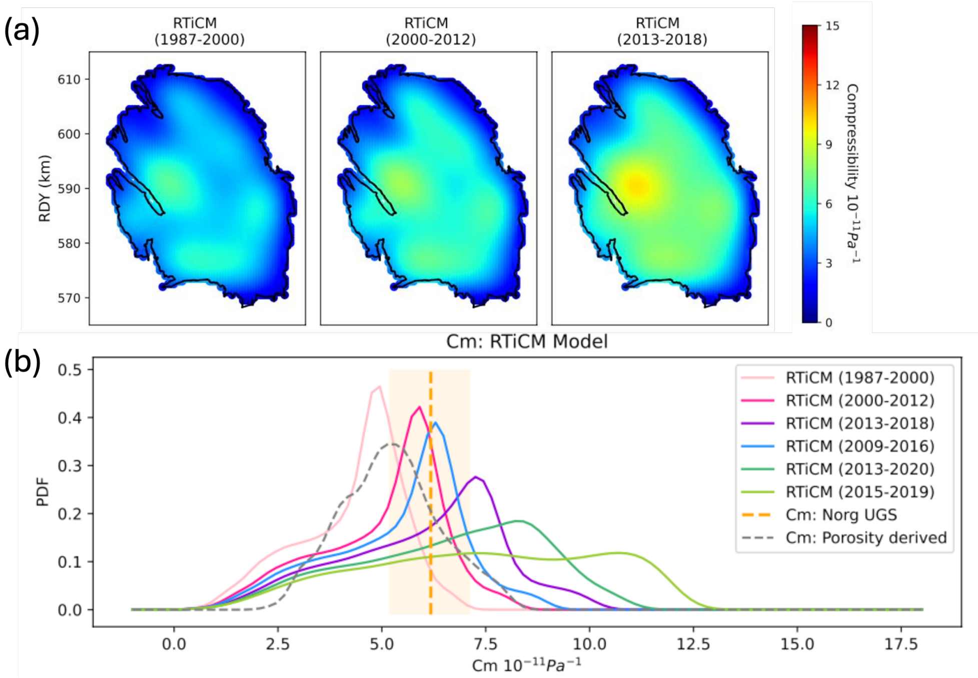

Figure 9: Apparent increase of elastic uniaxial compressibility (c'_m) in the Groningen field predicted with RTiCM. The model predicts an increase with time of the apparent uniaxial compressibility similar to the one inferred from the measurements (compare with Figure 4). (a) spatial distribution of c'_m inverted from simulations using RTiCM rheology in the field spanning various time intervals shown in title. (b) Probability density distribution of c'_m inverted from these simulations. The gray dashed line shows the porosity derived compressibility (NAM, 2013). The best-fitting compressibility at Norg is indicated by the orange dashed line, with its uncertainties represented by the shaded orange area.

Our analysis demonstrates that the RTiCM model has actually more merits than acknowledged in the reviews of the TNO report. First and foremost, it succeeds in providing a consistent framework to model data from laboratory experiments on rock samples from the Rotliegend reservoir, and subsidence data collected over the decades of exploitation of the Groningen gas field. Second the model allows to explain the purely elastic response observed at the Norg gas storage and the modest, but significant amount of inelastic compaction of the Groningen gas field. The surface deformation associated with the two gas fields are indeed reproduced relatively well by incorporating the RTiCM deformation in the GMG workflow and using the set of parameters proposed in the TNO report. So the model passes the up-scaling test relatively well (steps 4 and 5 in the logical sequence of Figure 1).

5 Conclusions

Our general conclusions are as follows:

1. The observation of concavity in subsidence vs pressure depletion is not sufficient to conclude for a delayed compaction and infer inelastic compaction (see Figure 2, and section 1).
2. Different fields with similar rock characteristics can show or not evidence of inelastic compaction. In particular, we do not find any evidence of inelastic compaction in the case of the Norg gas field (Figure 3). In contrast, in the case of the Groningen gas field, we do find conclusive evidence of inelastic compaction through temporal evolution of the apparent elastic compressibility field (Figure 4). The contrasted behavior observed at the two gas fields is consistent with our modeling results assuming a RTiCM with model parameters close to those proposed in NAM (2016b); TNO (2022) (Figure 9). This

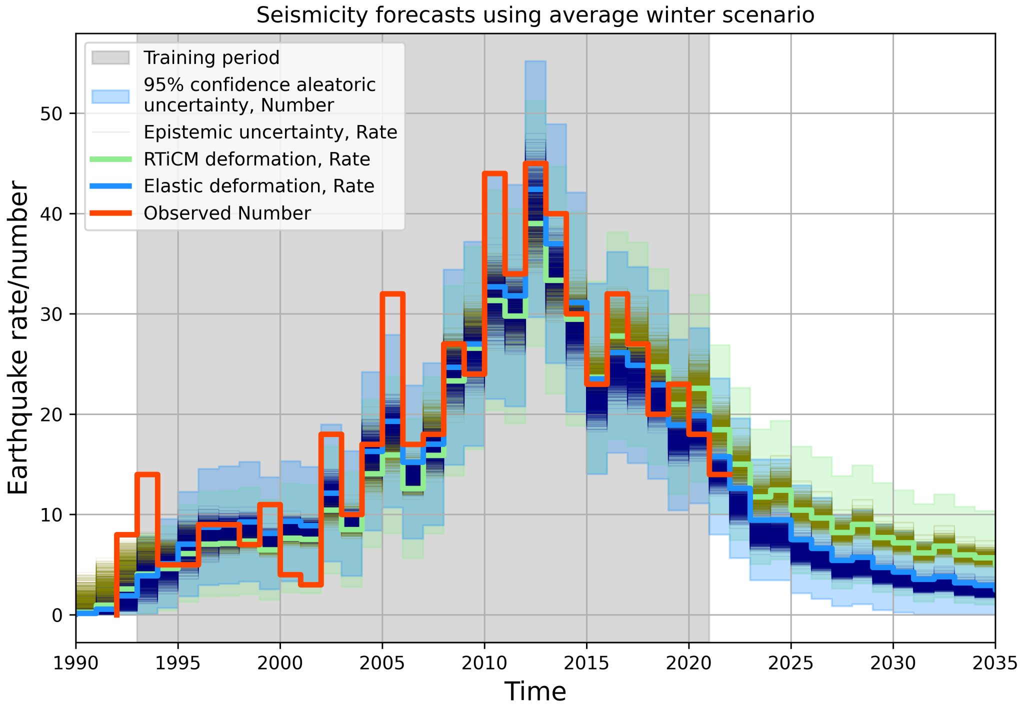

Figure 10: Comparison of observed seismicity with predicted seismicity assuming linear poroelastic compaction (blue) or RTiCM (green) . Orange curve shows the observed annual number of earthquakes using a completion magnitude of 1.2. The blue lines represent the 1000 best models (the best model in light blue) inverted using a linear elastic rheology for the reservoir and the over-and-underburden (see Appendix for details). The green lines represent the 1000 best models (the best model in light green) after inversion of the seismicity model using reservoir compaction calculated with the RTiCM parameters of (TNO, 2022). Blue and green lines are shown with their epistemic uncertainty (Kaveh et al., 2023). Aleatoric uncertainties on the best model are shown as shaded areas (Kaveh et al., 2023). The RTiCM model results in a longer tail of seismicity beyond the period of production.

does not mean that the RTiCM model is the unique model that can explain inelastic compaction, but it is one model that can.

3. Forecasts of surface subsidence (Figure 8) can be significantly affected by accounting for inelastic compaction. Careful calibration of the compressibility field using early time periods is a robust method for detecting inelastic compaction. It is probably difficult to reach a clear conclusion about about the rate dependence unless we are able to compute the virgin field compressibility and stressing rate (before any operations took place), and compare it with the production compressibility and stressing rate.

4. Simpler models (TD or SLS) which assume one single characteristic relaxation time, considered to be a material property, can be rejected. Such models would not be able to explain simultaneously the observations from Norg and Groningen gas fields and from the laboratory experiments (non-parallel isotachs). As pointed out in the Lehner report, a series of such models with a wide a range of relaxation times would be needed. The merit of developing such models is not clear to us.

5. An improved model assuming compaction driven by frictional grain boundary sliding could probably be developed that would take advantage of the progress made in understanding friction since RTCM was first formulated (de Waal, 1986). We expect that such a model would turn out very similar to RTiCM but could be more general, allowing for small inelastic strain modeling in 3-D (and could

account for the effect of temperature or fluid chemistry). First steps toward such a model have been proposed recently (Pijnenburg and Spiers, 2020). Alternative models of compaction could be developed assuming inelastic compaction of interstitial clay particles (Hol et al., 2018; Pijnenburg et al., 2019a,b; Pijnenburg and Spiers, 2020) or assuming brittle creep (Heap et al., 2009; Brantut et al., 2013, 2014). Such endeavors would require a combination of specific laboratory experiments, and numerical and analytical modeling informed with geodetic observations.

6. The effect of inelastic compaction on seismicity (Figure 10) is small over the calibration period because the difference with the prediction based on a linear poro-elastic model is compensated by the calibration of the seismicity model. However the models yield different predictions going forward into the future. The RTiCM model predicts a slower decay of the seismicity rate but the seismicity compared to the prediction of the linear poroelastic model is only 2% higher. Because the characteristic time of relaxation of the RTiCM model is inversely proportional to the stressing rate (Lehner report pp 25-26), the seismicity tail is probably fatter than would be predicted by any model of inelastic compaction where the relaxation time would be a material property. The RTiCM might therefore be seen is a relatively conservative model for hazard forecasts.

A Appendix: Methodologies developed at Caltech's GMG

The general workflow for seismicity forecasting in Groningen uses three different models. 1) a reservoir model that computes the space-time evolution of pressure changes in the field due to gas extraction. 2) a geomechanical model that relates the the computed field of fluid pressure depletion to reservoir strain. Here the reservoir deformation can be elastic or inelastic depending on the chosen rheology. And 3) a seismicity model that relates stress changes computed around the reservoir to seismicity rate changes. Here the focus is on the geomechanical model that relates the pressure changes (Δp), to the reservoir's uniaxial compaction (c_m) such that the total strain (ϵ_z) in the reservoir writes:

$$\epsilon_z(x, y, t) = f(c_m(x, y), \Delta p(x, y, t) \tag{5}$$

where f is a function that links the compressibility to the pressure changes and take the form of elastic ($\epsilon_z = \epsilon_{elastic}$) or inelastic rheologies (for the RTiCM as proposed by (Pruiksma et al., 2015), $\epsilon_z = \epsilon_d + \epsilon_s$). Then, the workflow allows to calibrate such function $f(c_m(x, y))$ to minimize the misfit with observed surface displacement. Once the model is calibrated, we can forecast surface displacement outside of the calibration period.

For more details on the use of different reservoir models, the reader is referred to (Smith et al., 2019; Meyer et al., 2022; Acosta et al., 2023). For more details on the inversion of elastic compressibility and the potential to use inelastic rheologies, the reader is referred to (Smith et al., 2019; Li et al., 2024). For further details on the methods to estimate seismicity from stress changes and the sampling procedures, the reader is referred to (Smith et al., 2022; Heimisson et al., 2022; Acosta et al., 2023; Kaveh et al., 2023).

Below is a brief overview of the different modules (modified from Acosta et al. (2023)).

A.1 From Fluid Extraction to Pressure Changes

A simplified reservoir model, assuming vertical flow equilibrium (VFE), is employed to calculate fluid pressure diffusion based on extraction history. The VFE assumption is valid under certain conditions, allowing the problem to be treated in two dimensions. In the case of the Groningen reservoir, this assumption holds well, supported by the reservoir's dimensions and permeabilities. The model assumes homogeneous transport properties and smooth reservoir thickness variation. The governing equation is solved using finite element methods, and model parameters are optimized through history matching of well pressure data, minimizing errors between simulated and observed pressures.

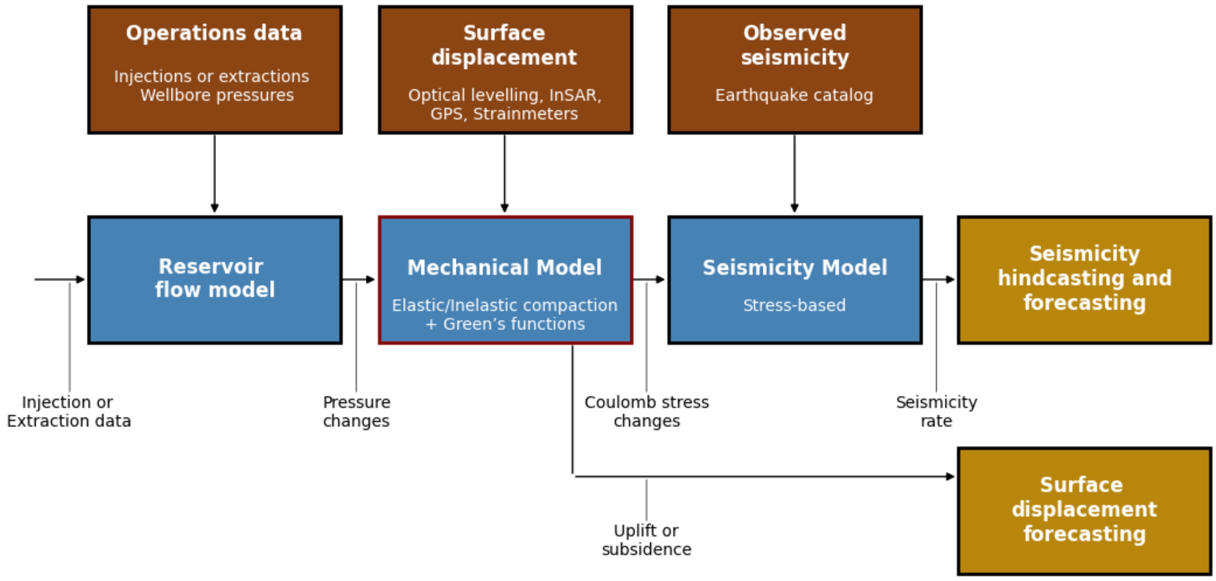

Figure A.1: Integrated modeling workflow for deformation and seismicity forecasting, FLOW2QUAKE (modified from (Acosta et al., 2023)). The workflow allows modular simulation of fluid pressure due to reservoir operations (injection or extraction), the resulting geomechanical deformation, and eventual induced seismicity. The blue boxes represent the different modules, that are calibrated using observational data (red boxes). The current article focuses on the development of our data-ready module for multiphase flow modeling of fluid injections or extractions in/from subsurface porous reservoirs. Note that in this work we allow different reservoir rheologies in the mechanical model (elastic or inelastic), and combine that with an elastic overburden to compute surface displacement.

A.2 From Pressure Changes to Reservoir Deformation and Stress Changes

We use a poroelastic mechanical model to translate fluid pressure changes into stress variations within and outside the reservoir. Reservoir compaction is modeled by discretizing the reservoir and applying a Green's function approach to relate compaction to stress and displacement (Geertsma, 1973; Kuvshinov, 2008; Smith et al., 2019). Stress fields are spatially smoothed, and Coulomb stress changes are computed, considering the fault's orientation and friction properties. The analysis highlights areas where stress changes are significant and notes potential overestimation due to simplifying assumptions (Smith et al., 2022).

A.3 From Stress Changes to Seismicity Rate Changes

The relationship between Coulomb stress changes and seismicity rates is modeled using a Threshold Rate and State failure function (Heimisson et al., 2022), allowing for time-dependent seismicity nucleation. An ensemble Markov Chain Monte Carlo algorithm is used to sample the probability distribution of model parameters, with misfit quantified through a Poisson or Gaussian log-likelihood function (Kaveh et al., 2023).

This workflow provides an efficient approach to forecasting and understanding the complex interplay between fluid extraction, reservoir deformation, and seismicity.

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